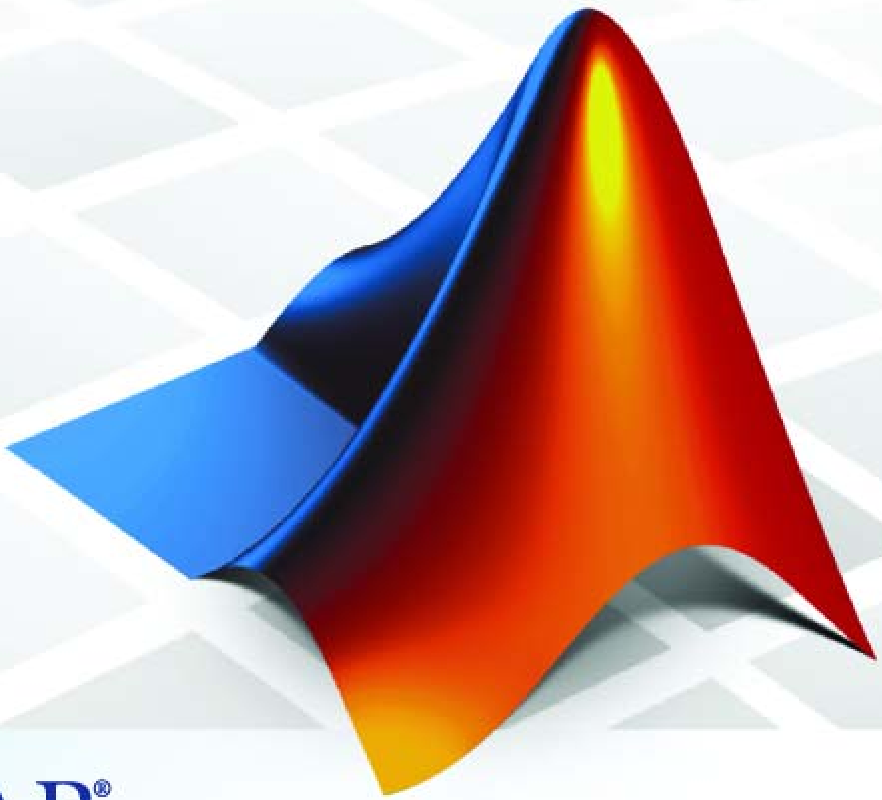
Getting Started with System Identification Toolbox 7

Lennart Ljung



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Getting Started with System Identification Toolbox

Revision History

March 2007	First printing	New for Version 7.0 (Release 2007a)
September 2007	Second printing	Revised for Version 7.1 (Release 2007b)

About the Developers

System Identification Toolbox is developed in association with the following leading researchers in the system identification field:

Lennart Ljung. Professor Lennart Ljung is with the Department of Electrical Engineering at Linköping University in Sweden. He is a recognized leader in system identification and has published numerous papers and books in this area.

Qinghua Zhang. Dr. Qinghua Zhang is a researcher at Institut National de Recherche en Informatique et en Automatique (INRIA) and at Institut de Recherche en Informatique et Systèmes Aléatoires (IRISA), both in Rennes, France. He conducts research in the areas of nonlinear system identification, fault diagnosis, and signal processing with applications in the fields of energy, automotive, and biomedical systems.

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Contents

6 Tutorial: Estimating Nonlinear Black-Box Models

Introduction to System Identification Toolbox

What Is System Identification Toolbox?

In this section...

What You Can Accomplish Using This Toolbox

System Identification Toolbox extends the MATLAB® computational environment for estimating linear and nonlinear mathematical models to fit measured data from dynamic systems.

System identification is especially useful for modeling systems that you cannot easily represent in terms of first principles. In this case, you use System Identification Toolbox to perform *black-box modeling*, where the measured data determines the model structure. Examples of complex dynamic systems requiring black-box models include engine subsystems, flight dynamics systems, thermofluid processes, and electromechanical systems.

You can also use System Identification Toolbox to compute the coefficients of ordinary differential and difference equations for systems modeled from first principles. Such models are called *grey-box models*.

For real-time applications in adaptive control, adaptive filtering, or adaptive prediction, you can use System Identification Toolbox to perform recursive parameter estimation.

You can validate models directly after each estimation to help you select the best dynamic model for your system.

For an overview of using System Identification Toolbox, see "How to Use System Identification Toolbox" on page 1-9.

Types of Data You Can Model

For linear models, System Identification Toolbox supports both time- and frequency-domain data with single or multiple inputs and outputs. Time-domain data can be either real or complex.

For nonlinear models, this toolbox supports only time-domain data.

Time-domain data is one or more input variables *u(t)* and one or more output variables *y(t)*, sampled as a function of time. A special case of time-domain data is time-series data, which is one or more outputs *y(t)* and no measured input. *Frequency-domain data* is the Fourier transform of the input and output time-domain signals. *Frequency-response data*, also called *frequency-function data*, represents complex frequency-response values for a linear system characterized by its transfer function G.

You can measure frequency-response data values directly using a spectrum analyzer, for example. In this section, *frequency-domain* and *frequency-response* are both referenced as frequency-domain data for the sake of brevity.

For time-series data, you can estimate both linear and nonlinear models.

Stages of Identifying Dynamic Systems

The general system identification process might include the following stages:

1. Experimental design and data acquisition
2. Data analysis and preprocessing, including plotting the data, removing offsets and linear trends, filtering, resampling, and selecting regions of interest
3. Estimation and validation of models
4. Model analysis and transformation, such as linear analysis, reducing model order, and converting between discrete-time and continuous-time representations.
5. Model usage for intended applications, such as simulation or prediction of output values or control design

System Identification Toolbox supports all of these stages except data acquisition. This toolbox provides some support for experimental design by enabling you to generate input signals with different properties. You can also model data to validate and refine your experimental design.

Using Estimated Models

You can use System Identification Toolbox commands to simulate or predict model output. You can also import models into Simulink®.

In control design applications, you might use System Identification Toolbox to model a plant for control design. For example, you can import your linear plant model into Control System Toolbox, Model Predictive Control Toolbox, and Robust Control Toolbox.

Related Products

The following table summarizes the products that extend and complement System Identification Toolbox. For current information about these and other MathWorks products, point your Web browser to:

```
www.mathworks.com
```

Products That Extend System Identification Toolbox

Product	Description
Control System Toolbox	Provides extensive tools to analyze plant models created in System Identification Toolbox and to tune control systems based on these plant models.
Model Predictive Control Toolbox	Uses the linear plant models created in System Identification Toolbox for predicting plant behavior that is optimized by the model-predictive controller.

Products That Extend System Identification Toolbox (Continued)

Product	Description
Neural Network Toolbox	Provides flexible neural-network structures for estimating nonlinear models using System Identification Toolbox.
Optimization Toolbox	When this toolbox is installed, you have the option of using the `lsqnonlin` optimization algorithm for nonlinear identification.
Robust Control Toolbox	Provides tools to design multiple-input and multiple-output (MIMO) control systems based on plant models created in System Identification Toolbox. Robust Control Toolbox lets you assess robustness based on confidence bounds for the identified plant model.

Products That Extend System Identification Toolbox (Continued)

Product	Description
Signal Processing Toolbox	Provides additional options for: • Filtering (System Identification Toolbox provides only the fifth-order Butterworth filter.) • Spectral analysis After using the advanced data processing capabilities of Signal Processing Toolbox, you can import the data into System Identification Toolbox for modeling.
Simulink	Provides System Identification blocks for simulating the models you identified using System Identification Toolbox. Also provides blocks for model estimation.

Starting System Identification Toolbox

After you have installed System Identification Toolbox, you can use either the System Identification Tool GUI or the command-line syntax.

To open the System Identification Tool GUI:

- Select **Start > Toolboxes > System Identification** from the MATLAB Command Window.

Alternatively, you can open the System Identification Tool GUI by typing the following command at the MATLAB prompt:

```
ident
```

To use the toolbox commands, type the commands directly in the MATLAB Command Window. For more information about the commands, see the corresponding reference pages.

For information about whether to use the GUI or the command line, see “When to Use the GUI Versus the Command Line” on page 1-8.

When to Use the GUI Versus the Command Line

New users should start by using the System Identification Tool GUI to become familiar with the product.

You can work either in the GUI or at the command line to preprocess data, and estimate, validate, and compare models.

For a tutorial that walks you through typical command-line tasks, see Chapter 5, "Tutorial: Estimating Linear Models Using the Command Line".

However, the following operations are available only at the command line:

- Generating input and output data (see `idinput`).
- Estimating coefficients of linear and nonlinear ordinary differential or difference equations (grey-box models).
- Using recursive online estimation methods. See the topics on estimating linear models recursively in the *System Identification Toolbox User's Guide*.
- Converting between continuous-time and discrete-time (see `c2d` and `d2c`).
- Converting models to LTI objects (see the `ss`, `tf`, and `zpk` reference pages).

Note Conversions to LTI objects require Control System Toolbox.

Tip To learn more about estimating and validating models at the command line, see Chapter 5, "Tutorial: Estimating Linear Models Using the Command Line".

How to Use System Identification Toolbox

System identification is an iterative process, where you estimate many different models for your data and compare model performance. Ultimately, you choose the simplest model that best describes the dynamics of your system.

Because System Identification Toolbox lets you estimate different model structures quickly, you should try many different structures to see which one gives the best results.

A system identification workflow might include the following tasks:

1. Prepare data for system identification by:
   - Importing data into the MATLAB Workspace browser.
   - Importing data into the System Identification Tool GUI or creating an `iddata` or `idfrd` object in the MATLAB Command Window, depending on which environment you are using.
   - Plotting data to examine both time- and frequency-domain behavior.

     To analyze the data for the presence of constant offsets and trends, delay, feedback, and signal excitation levels, you can use the `advice` command.
   - Preprocessing data by removing offsets and linear trends, interpolating missing values, filtering to emphasize a specific frequency range, or resampling using a different time interval.
2. Estimate linear or nonlinear models:
   - Time-domain correlation analysis models
   - Frequency-response models using spectral analysis
   - Low-order transfer functions (process models)
   - Polynomial models
   - State-space models
   - Nonlinear black-box models
   - User-defined (grey-box) models

3. Validate models.

   When you do not achieve a satisfactory model, try a different model structure and order or try another identification algorithm.

   You might need to preprocess your data before doing further estimation. For example, if there is too much high-frequency noise in your data, you might need to filter or decimate (resample) the data before modeling.

4. (Optional) Transform model representation by:

   - Transforming between discrete-time and continuous-time representation.
   - Transforming between frequency-response, state-space, and polynomial forms.
   - Reducing model order based on pole-zero cancellations.
   - Linearizing a nonlinear plant. For more information about these functions, see the `lintan` and `linapp` reference pages.

5. Simulate or predict model output.

6. Design a controller for the estimated plant using other MathWorks products.

   You can import an estimated linear model into Control System Toolbox, Model Predictive Control Toolbox, Robust Control Toolbox, or Simulink for controller design.

Accessing the Documentation and Demos

In this section…

Accessing Documentation

MathWorks technical documentation is available online from the **Help** menu from the MATLAB desktop.

System Identification Toolbox documentation contains the following components:

- Getting Started — Provides essential information for mapping your problem to the capabilities of System Identification Toolbox. Step-by-step tutorials walk you through the most common system identification tasks.
- User's Guide — Fully describes the System Identification Toolbox tasks.
- Reference — Describes the essential syntax and usage of System Identification Toolbox objects, methods, and functions.
- Release Notes — Describes important changes in the current version of System Identification Toolbox and compatibility considerations.

New Users. If you are new to System Identification Toolbox, this Getting Started documentation will help you begin using System Identification Toolbox quickly. After a brief introduction to the types of models you can estimate, follow the steps in the tutorials to estimate models in the System Identification Tool graphical user interface (GUI) or the MATLAB Command Window.

Experienced Users. If you are familiar with System Identification Toolbox, search or browse the documentation for information about specific tasks.

Accessing Demos

System Identification Toolbox provides demo files that show you how to estimate models for dynamic systems from measured data. The available demos include both case studies and tutorials.

To access the demos in the Help browser, type the following command at the MATLAB prompt:

```
demo
```

In the **Demos** pane, select **Toolboxes > System Identification** to open the list of available demos.

Learning More

The goal of the System Identification Toolbox documentation is to provide you with the necessary information to use this product. Additional resources are available to help you learn more about specific aspects of system identification theory and applications.

The following book describes methods for system identification and physical modeling:

Ljung, L., and T. Glad. *Modeling of Dynamic Systems*. PTR Prentice Hall, Upper Saddle River, NJ, 1994.

These books provide detailed information about system identification theory and algorithms:

- Ljung, L. *System Identification: Theory for the User*. Second edition. PTR Prentice Hall, Upper Saddle River, NJ, 1999.
- Söderström, T., and P. Stoica. *System Identification*. Prentice Hall International, London, 1989.

For information about working with frequency-domain data, see the following article:

Pintelon, R., and J. Schoukens. *System Identification. A Frequency Domain Approach*. IEEE Press, New York, 2001.

For more information about systems and signals, see the following book:

Oppenheim, J., and Willsky, A.S. *Signals and Systems*. PTR Prentice Hall, Upper Saddle River, NJ, 1985.

The following textbook describes numerical techniques for parameter estimation using criterion minimization:

Dennis, J.E., Jr., and R.B. Schnabel. *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. PTR Prentice Hall, Upper Saddle River, NJ, 1983.

Choosing Models to Estimate

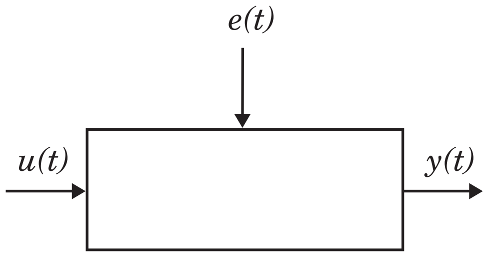
About Models

In this section...

What Is a Model?

A model is a tool you use to answer questions about the system without having to perform experiments. For example, you might use a model to simulate the output of a system for a given input and analyze the system response. Alternatively, you might be interested in using the model to predict future output of a system based on previous inputs and outputs.

Models describe the relationship between one or more measured input signals, *u(t)*, and one or more measured output signals, *y(t)*. Input and output signals can be measured in the time or frequency domain.



In real systems, there are additional inputs that affect the system output and that you cannot measure or control. Such unmeasured inputs are called *disturbances* or *noise*, *e(t)*. For example, if the system is an airplane, its inputs might be the positions of various control surfaces, such as ailerons and elevators. The system outputs might be the airplane orientation, velocity, and position. The noise might be turbulence and wind gusts that affect the outputs.

You get the best results when you estimate models using two independent data sets: one data set for estimation (*estimation data*), and the other data set for validation (*validation data*).

Categories of Models

- "User-Defined (Grey-Box) Models" on page 2-4
- "Black-Box Models" on page 2-4
- "Continuous-Time Models" on page 2-6
- "Discrete-Time Models" on page 2-6

User-Defined (Grey-Box) Models

Grey-box models are differential or difference equations that you construct from first principles based on physical insight into the system. In this case, you already know the model structure and estimate its parameters.

Black-Box Models

Black-box models are flexible mathematical structures that are not based on first principles. You can estimate both nonparametric and parametric black-box models. In case of parametric models, the data helps you to both select the best model structure and estimate the parameters.

System Identification Toolbox supports the following types of black-box models:

- Nonparametric models
- Parametric models

Nonparametric Models. *Nonparametric models* are black-box models that consist of data tables representing the impulse response, step response, and frequency response of the system. Because nonparametric models are not represented by a compact mathematical formula with adjustable parameters, such models do not impose a specific mathematical structure on your system.

Nonparametric models serve well as preliminary models that you can use to analyze system characteristics. For example, estimating the transient response provides insight into the rise time and settling time of the system response. Similarly, estimating frequency response might indicate the order of the system, locations of resonances and notches, crossover frequencies, and the bandwidth of the system.

You can estimate nonparametric models using the following methods:

- *Correlation analysis* estimates the impulse or step response of the system, also called *transient response*.
- *Spectral analysis* estimates the frequency response of the system.

  Frequency response describes the steady-state response of a system to sinusoidal inputs. For a linear system, a sinusoidal input of a specific frequency results in an output that is also a sinusoid with the same frequency, but with a different amplitude and phase. The frequency response function describes the amplitude change and phase shift as a function of frequency. In other words, the frequency response function is the Laplace transform of the impulse response that is evaluated on the imaginary axis. You can use a Bode plot to visualize the frequency response of the system, which shows the amplitude change and the phase shift as a function of the sinusoid frequency.

Parametric Models. *Parametric models* are black-box models that have a well-defined mathematical structure, and this structure is fit to the time- and frequency-domain data by adjusting the coefficient values, or *model parameters*.

System Identification Toolbox supports several linear and nonlinear model structures.

The simplest parametric models are linear, polynomial, discrete-time difference equations. For example, the following equation represents the ARX polynomial structure:

$$\begin{aligned} y(t) + a_1 y(t-T) + a_2 y(t-2T) = \\ b_1 u(t-T) + b_2 u(t-2T) \end{aligned}$$

y(t) is the output, *u(t)* is the input, and T is the sampling interval.

Other supported linear parametric models include ARX, ARMAX, Box-Jenkins (BJ), output-error (OE), and state-space models. One important difference between these models is the way they model noise.

In case of nonlinear black-box models, you can estimate nonlinear ARX and Hammerstein-Wiener structures.

Continuous-Time Models

A *continuous-time model* describes the relationship between continuous-time input and output signals. You typically represent continuous-time systems using differential equations.

System Identification Toolbox lets you estimate linear continuous-time models directly. For example, you can estimate low-order transfer functions, called *continuous-time process models*, from time- or frequency-domain data. You can also estimate a continuous-time model of any structure from frequency-domain data.

Discrete-Time Models

In practice, input and output signals are collected at specific time intervals, or *samples*. A *discrete-time model* expresses the relationship between the values of the signals at the sampling instants. Such models are typically described by difference equations.

Supported Models

System Identification Toolbox supports the following types of linear and nonlinear models:

- Nonparametric models, including transient-response and frequency-response models
- Linear parametric models:
  - Low-order, continuous-time transfer functions (process models)
  - Polynomial models, including ARX, ARMAX, Box-Jenkins, and output-error models
  - Linear state-space models

- Nonlinear black-box models
- Grey-box models, represented by linear or nonlinear ordinary differential equations or ordinary difference equations
- Time-series models

Which models you estimate depends on the nature of the dynamic system, on the type of behavior that you expect, and on the intended use of the model.

In some cases, a specific mathematical form is preferable because the estimated parameters have a physical interpretation.

If you require estimates of dynamic characteristics without detailed parametric models, you can use nonparametric models.

Mathematical Description of Dynamic Models

As you work with System Identification Toolbox, you estimate models that are special cases of the following general mathematical description of dynamic systems:

$$y(t) = g(u, \theta) + v(t)$$

The output *y(t)* of a system is determined by *g*, which might be a function of all previous inputs *u(s)* ($s \le t$) and system parameters θ . *v(t)* is the output noise.

For nonlinear models, *g* can take a variety of forms.

For linear models, the general symbolic model description is given by:

$$y = Gu + He$$

G is an operator that describes the system dynamics from the input to the output. *G* is often called a *transfer function* between *u* and *y*. *H* is an operator that describes the properties of the additive output disturbance and is called a *disturbance model*, or *noise model*.

When to Estimate Black-Box Models

System identification is especially useful for modeling systems that you cannot easily represent in terms of first principles or known physical laws. In this case, System Identification Toolbox uses measured data to help determine the structure of the resulting black-box model. The parameters of a black-box model might not have a physical interpretation.

You can estimate several black-box models that have different orders and choose the model that has the best performance. Black-box models can be linear or nonlinear models. Black-box models can also be continuous-time or discrete-time models.

Black-box modeling has the following advantages:

- You do not need to know the structure and order of your model to get started quickly.
- You can estimate many model structures and compare them to choose the best one.

Black-box model structures differ based on how they model noise. For example, some structures, such as the Box-Jenkins (BJ) structure, decouple the dynamics from the noise.

Tip Unless you have specific requirements about how to handle noise or insight into how noise affects your system, you do not need to worry about which structure is correct. Instead, you can estimate different structures and use the validation process to identify a model with the best performance.

For more information about modeling noise, see “Estimating Noise Models” on page 2-17.

When to Estimate Models from First Principles

If you understand the physics of your system and can represent the system using an ordinary differential or difference equation (ODE), then you can use System Identification Toolbox to perform grey-box modeling.

A *grey-box model* has a known mathematical structure and unknown parameters. You capture the ODE and the parameters you want to estimate in an M-file or MEX-file, and use System Identification Toolbox to estimate the model parameters.

Grey-box modeling has the following advantages over black-box modeling:

- You can impose known constraints on model characteristics, such as model parameters and noise variance.
- There are potentially fewer parameters to estimate.
- You can specify couplings between parameters when defining the model structure.
- In the nonlinear case, you can specify the dynamic equations explicitly.

Grey-box modeling is preferred. However, grey-box modeling requires that you know the relationship between the system variables and the parameters, which can be time consuming.

For detailed information about grey-box modeling, see the topics on estimating linear and nonlinear grey-box models in the *System Identification Toolbox User's Guide*.

When to Estimate Linear Versus Nonlinear Models

System Identification Toolbox lets you estimate both linear and nonlinear models. In practice, all systems are nonlinear and the output is a nonlinear function of the input variables. However, a linear model is often sufficient to accurately describe the system dynamics.

For a grey-box model, its linear or nonlinear structure is set by its differential or difference equations.

For a black-box model, you can choose whether to estimate linear or nonlinear models. Linear approximations are very useful because they are simple and provide good results in many situations. Therefore, always estimate linear models first and see how well these models represent the dynamics.

Note For nonlinear black-box models, you can only estimate discrete-time models using time-domain data.

Follow these guidelines to estimate nonlinear black-box models:

- When you have physical insight that the system is nonlinear, try transforming your input and output variables such that the relationship between the transformed variables is linear.

  For example, you might be dealing with a process that has current and voltage as inputs to an immersion heater, and the temperature of the heated liquid as an output. In this case, the output depends on the inputs via the power of the heater, which is equal to the product of current and voltage. Instead of fitting a nonlinear model to two-input and one-output data, you can create a new input variable by taking the product of current and voltage and then fitting a linear model to the single-input/single-output data.

- You plot the response of the system to a specific input and notice that the responses are different depending on the input level or input sign. For example, you might see that the output response to an input step up is much faster than the response to a step down. This response behavior indicates that the system is nonlinear and you need a nonlinear model.

- You try many linear models of varying complexity, but the model output does not adequately reproduce the measured output. This inability of the model to reproduce measured output might be caused by noisy data or by nonlinear system behavior.

Choosing Models Based on Available Data

In this section...
"Supported Models for Time-Domain Data" on page 2-12
"Supported Models for Frequency-Domain Data" on page 2-13

Supported Models for Time-Domain Data

Continuous-Time Models

To get a linear, continuous-time model of arbitrary structure for time-domain data, you can estimate a discrete-time model, and then use `d2c` to transform it to a continuous-time model.

However, you can estimate the following types of continuous-time models directly:

- Low-order transfer functions (process models)
- Linear polynomial models, including ARX and output-error (OE) models
- State-space models

Discrete-Time Models

You can estimate any linear and nonlinear discrete-time model supported by System Identification Toolbox.

Grey-Box Models

For linear and nonlinear grey-box models, you can estimate both continuous-time and discrete-time models from time-domain data.

Nonlinear Models

You can estimate all supported discrete-time nonlinear black-box models for time-domain data, which includes Hammerstein-Wiener and nonlinear ARX models.

You can also estimate nonlinear grey-box models for time-domain data.

Supported Models for Frequency-Domain Data

Frequency-domain data comes in two types:

- Continuous-time data
- Discrete-time data

To designate discrete-time data, you must set the sampling interval of the data to the experimental data sampling interval. To designate continuous-time data, you must set the sampling interval of the data to zero.

You can set the data sampling interval when you import the data set into the System Identification Tool GUI, or when you create the data object at the command line. Setting the sampling interval to zero corresponds to taking a Fourier transform of continuous-time data.

Continuous-Time Models

To get a linear, continuous-time model of arbitrary structure for time-domain data, you can estimate a discrete-time model, and then use `d2c` to transform it to a continuous-time model.

You can estimate the following types of continuous-time models directly:

- Low-order transfer functions (process models)
- Linear polynomial models, including ARX and output-error (OE) models
- State-space models

  From continuous-time frequency-domain data, you can estimate continuous-time state-space models. From discrete-time frequency-domain data, you can estimate continuous-time black-box models with canonical parameterization.

Discrete-Time Models

You can estimate only ARX and output-error (OE) polynomial models using frequency-domain data in System Identification Toolbox. Other model structures include noise models, which are not supported for frequency-domain data.

Grey-Box Models

For linear grey-box models, you can estimate both continuous-time and discrete-time models from frequency-domain data.

Nonlinear grey-box models are supported only for time-domain data.

Nonlinear Models

Frequency-domain data is not relevant to nonlinear models. Thus, nonlinear models support only time-domain data.

Supported Continuous-Time and Discrete-Time Models

For linear and nonlinear grey-box models, you can specify any ordinary differential or difference equation to represent your continuous-time or discrete-time model, respectively. In the linear case, both time-domain and frequency-domain data are supported. In the nonlinear case, only time-domain data is supported.

The following tables summarize supported continuous-time and discrete-time parametric models.

Supported Continuous-Time Models

Model Type	Description
Low-order transfer functions (process models)	Estimate low-order transfer functions (up to three free poles) from either time- or frequency-domain data.
Linear, black-box polynomial models: • ARX • ARMAX • Output-error • Box-Jenkins	To get a linear, continuous-time model of arbitrary structure from time-domain data, you can estimate a discrete-time model, and then use `d2c` to transform it into a continuous-time model. For frequency-domain data, you can directly estimate only the ARX and output-error (OE) continuous-time models. Other structures include noise models and are not supported for frequency-domain data.
State-space models	To get a linear, continuous-time model of arbitrary structure for time-domain data, you can estimate a discrete-time model, and then use `d2c` to transform it into a continuous-time model. For frequency-domain data, you can estimate continuous-time state-space models directly.
Linear grey-box models	Estimate ordinary differential equations (ODEs) from either time- or frequency-domain data.
Nonlinear grey-box models	Estimate arbitrary differential equations (ODEs) from time-domain data.

Supported Discrete-Time Models

Model Type	Description
Linear, black-box polynomial models: • ARX • ARMAX • Output-error • Box-Jenkins • State-space	Estimate arbitrary-order, linear parametric models from time- or frequency-domain data. To get a discrete-time model, your data sampling interval must be set to the (nonzero) value you used to sample in your experiment.
Nonlinear black-box models: • Nonlinear ARX • Hammerstein-Wiener	Estimate from time-domain data only.
Linear grey-box models	Estimate ordinary difference equations from time- or frequency-domain data.
Nonlinear grey-box models	Estimate ordinary difference equations from time-domain data.

Estimating Noise Models

In this section...

What Is a Noise Model?

For linear models, the general symbolic model description is given by the following equation:

$$y = Gu + He$$

G is an operator that describes the system dynamics from the input to the output. e is an unmeasured input that is the noise source. H is an operator that describes how the system forms the additive noise from e and is called a *disturbance model*, or *noise model*.

In this description of the additive noise, the noise source e is *white noise*, which means that it is entirely unpredictable. In other words, it is impossible to guess the value of $e(t)$ regardless of how accurately you measured the past data up to time t-1.

The actual disturbance contribution to the output, He, has real significance and contains all the known and unknown influences on the measured y not included in the input u. Therefore, if you repeat and experiment with the same input, He explains why the output signal is typically different.

The source of the noise, e, need not have a physical significance. In the case of an airplane, it is sufficient to estimate the noise in a black-box manner as arising from a white noise source via a transfer function H. Thus, you do not need to know how the wind gusts and turbulence are generated physically and all that matters are the characteristics of He, such the frequency content or spectrum of He.

When to Estimate a Noise Model

In the simplest case, you do not estimate a noise model. Instead, you can handle the additive noise term by setting $H = 1$, which corresponds to the noise source e affecting the output directly. The resulting model is called the *output-error model*.

Tip Omit estimating a noise model when you have a good signal-to-noise ratio (SNR). With a good SNR, information about G in the data is not corrupted.

You might choose to estimate a noise model in the following situations:

- You are specifically interested in a noise model, such as when developing noise-cancelation and noise-attenuation technologies, or for disturbance rejection in control design applications.
- You want to use the noise characteristics to improve the estimation of the dynamic model, G, by emphasizing the frequencies that are least affected by noise during the estimation.

Tip To see if estimating a noise model H might help you improve the dynamic model G, estimate models with and without noise and compare their simulated outputs.

Types of Model Structures That Support Noise Models

You can use System Identification Toolbox to estimate a deterministic noise model in addition to the dynamic model for linear model structures.

If you decide that a good noise model is important, choose the ARMAX, Box-Jenkins, or state-space model structures that provide additional parameters for modeling noise. For more information about these model structures, see the topic on estimation linear parametric models in the *System Identification Toolbox User's Guide*.

Output-error (OE) and ARX models are not sufficiently flexible for modeling noise. Output-error models produce a trivial noise model with H=1, and ARX models produce a noise model that is coupled to the dynamics.

Note Nonlinear ARX and Hammerstein-Wiener models do not support parametric noise models.

How Feedback Affects Model Choice

In this section...

Unreliable Models in the Presence of Feedback

When you estimate a model structure that includes a flexible noise model, the estimation methods work equally well for systems with and without feedback. *Feedback* means that your system operates in a closed loop and the past outputs affect the current inputs. Examples of structures with flexible noise models include ARMAX, Box-Jenkins (BJ), and state-space models.

However, the following models are unreliable when feedback is present in your system:

- Time-domain correlation-analysis models estimated using `cra`.

  If you estimate the impulse response using `impulse`, the response before time equal to `0` is caused by the feedback mechanism and does not represent system dynamics.

- Frequency-analysis models estimated using the `etfe`, `spa`, or `spafdr` spectral-analysis methods.
- State-spate models estimated using the noniterative estimation method `n4sid`.
- Model structures that have inaccurate noise models, such as output-error (OE) and state-space models with the property `DisturbanceModel` set to `None`. For more information, see the reference pages corresponding to these models.

Detecting Feedback in the Data

If you are unsure about the presence of feedback, use System Identification Toolbox to detect feedback in your data:

- Use the `advice` command on your data set. Also, you can use the `feedback` command to get detailed information about the nature of the feedback.

- Use the `impulse` command on your data set to plot the estimated impulse response. Significant values of the impulse response at negative lags might indicate feedback.
- On residual analysis plots, significant correlation between residuals and inputs at negative lags indicates feedback. For more information about residual analysis, see the corresponding section in the *System Identification User's Guide*.

Modeling Multiple-Output Systems

In this section...

Challenges of Modeling Multiple-Output Systems

Modeling multiple-output systems is more challenging because input/output couplings require additional parameters to obtain a good fit and more-complex models. In general, a model should be better when more inputs are included. Including more outputs typically leads to worse simulation results because it is more difficult to reproduce the behavior of several outputs.

Modeling Multiple Outputs Directly

You can estimate the following types of models for multiple-output data:

- Impulse- and step-response models using correlation analysis
- Frequency-response models using spectral analysis
- Linear ARX models or state-space models
- Nonlinear ARX and Hammerstein-Wiener models

State-space models provide the easier approach for estimating multiple-output models directly.

Modeling Multiple Outputs as a Combination of Single-Output Models

You may find that it is more difficult for a single model to explain the behavior of several outputs. If you get a poor fit estimating a multiple-output model directly, you can try building models for subsets of the most important input and output channels.

Use this approach when no feedback is present in the dynamic system and there are no couplings between the outputs. If you are unsure about the presence of feedback, see "Detecting Feedback in the Data" on page 2-20.

To construct partial models, use subreferencing to create partial data sets, such that each data set contains all inputs and one output. For more information about creating partial data sets, see the following sections in the *System Identification Toolbox User's Guide*:

- For working in the System Identification Tool GUI, see the topic about creating data sets from selected channels.
- For working at the command line, see the topic about subreferencing data objects.

After validating the single-output models, use vertical concatenation to combine these partial models into a single multiple-output model. For more information about concatenation, see the corresponding topic in the *System Identification Toolbox User's Guide*.

You can try refining the multiple-output model using the original (multiple-output) data set.

Tutorial: Estimating Linear Models Using the GUI

Exporting the Model to the MATLAB Workspace Browser (p. 3-49)	How to make the model available to operations in the MATLAB Command Window for further processing with this toolbox or other MathWorks products.
Exporting the Model to the LTI Viewer (p. 3-51)	How to export models to the LTI Viewer, which is available if you installed Control System Toolbox.

About This Tutorial

In this section...

Objectives

Estimate and validate linear models from single-input/single-output (SISO) data to find the one that best represents your system dynamics.

After completing this tutorial, you will be able to accomplish the following tasks using the System Identification Tool GUI:

- Import data arrays from the MATLAB Workspace browser into the GUI.
- Plot the data.
- Preprocess data by removing offsets from the input and output signals.
- Estimate, validate, and compare linear models.
- Export models to the MATLAB Workspace browser.

This tutorial is based on the example in *System Identification: Theory for the User*, Second Edition, by Lennart Ljung, Prentice Hall PTR, 1999.

Sample Data

The sample data is the MAT-file `dryer2.mat`, which contains single-input/single-output (SISO) time-domain data from Feedback Process Trainer PT326.The input and output signals each contain 1000 data samples.

This system heats the air at the inlet using a mesh of resistor wire, much like a hair dryer. The input is the power supplied to the resistor wires, and the output is the air temperature at the outlet.

Note The tutorial uses time-domain data to demonstrate how you can estimate linear models. The same workflow also applies to frequency-domain data.

Preparing Data

In this section...

Loading Data into the MATLAB Workspace Browser

Load sample data in `dryer2.mat` by typing the following command at the MATLAB prompt:

```
load dryer2
```

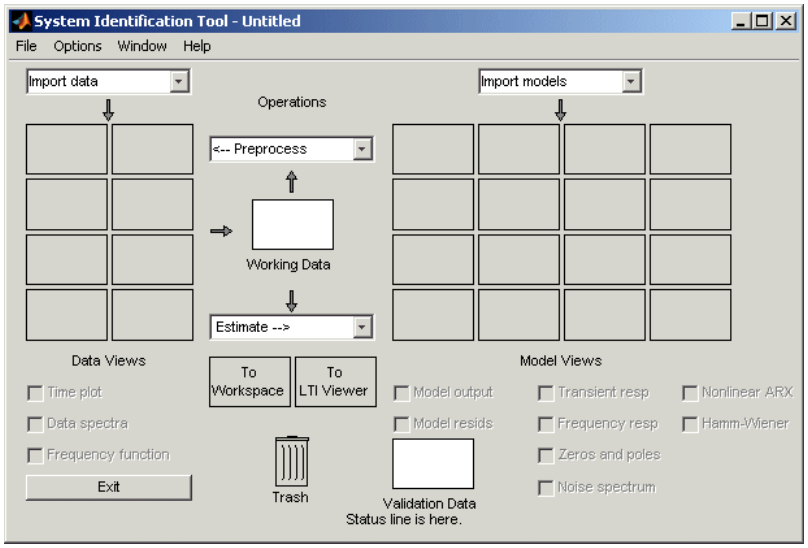
This command loads the data into the MATLAB Workspace browser as two column vectors, `u2` and `y2`, respectively. The variable `u2` is the input data and `y2` is the output data.

Opening the System Identification Tool GUI

To open the System Identification Tool GUI, type the following command at the MATLAB prompt:

```
ident
```

The default session name, `Untitled`, appears in the title bar.



Importing Data Arrays into the System Identification Tool

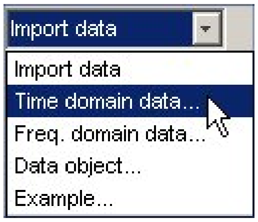
You can import the single-input/single-output (SISO) data from a sample data file `dryer2.mat` into the GUI from the MATLAB Workspace browser.

You must have already opened the System Identification Tool window, as described in "Opening the System Identification Tool GUI" on page 3-5.

Note The input and output signals need not have the same number of data samples.

---

1 In the System Identification Tool window, select **Import data > Time domain data**. This action opens the Import Data dialog box.

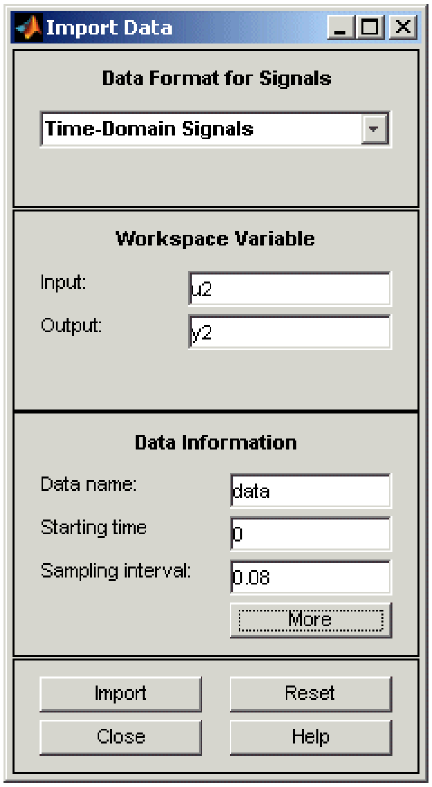


2 Specify the following options:

- **Input** — Enter `u2` as the name of the MATLAB variable that is the input signal.
- **Output** — Enter `y2` as the name of the MATLAB variable that is the output signal.
- **Data name** — Change the default name to `data`. This name labels the data in the System Identification Tool window after the import operation is completed.
- **Starting time** — Enter `0` as the starting time. This value designates the starting value of the time axis on time plots.
- **Sampling interval** — Enter `0.08` as the time between successive samples in seconds. This value is the actual sampling interval in the experiment.

Tip System Identification Toolbox uses the sampling interval during model estimation and to set the horizontal axis on time plots. If you transform a time-domain signal to a frequency-domain signal, the Fourier transforms are computed as discrete Fourier transforms (DFTs) using this sampling interval.

The Import Data dialog box now resembles the following figure.



Time-Domain Signals

Input: u2

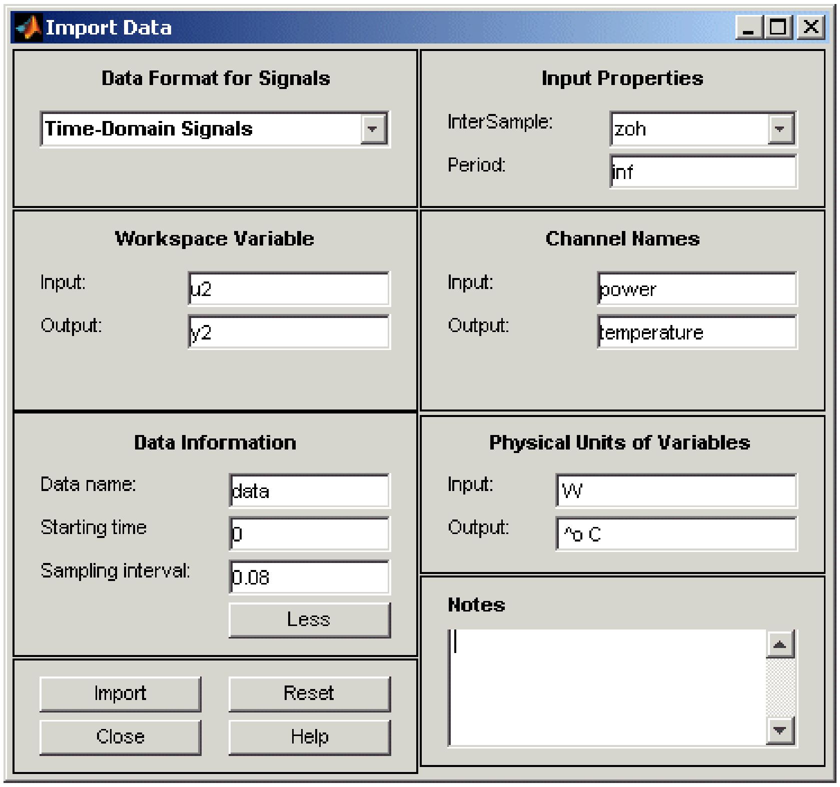
Output: y2

Data name: data

Starting time 0

Sampling interval: 0.08

3 In the **Data Information** area, click **More** to expand the dialog box. Enter the settings shown in the following figure.



Data Format for Signals: Time-Domain Signals

Input Properties
InterSample: zoh
Period: inf

Workspace Variable
Input: u2
Output: y2

Channel Names
Input: power
Output: temperature

Data Information
Data name: data
Starting time 0
Sampling interval: 0.08

Physical Units of Variables
Input: W
Output: ^o C

Input Properties

- **InterSample** — Enter `zoh` (zero-order hold) to maintain a piecewise-constant input signal between samples. This setting specifies the behavior of the input signals between samples when you transform the resulting models between discrete-time and continuous-time representations.

  Other possible settings include first-order hold (`foh`) and bandwidth-limited behavior (`bl`), where the latter specifies that the continuous-time input signal has zero power above the Nyquist frequency (equal to the inverse of the sampling interval).

  Note See the `d2c` and `c2d` reference page for more information about transforming between discrete-time and continuous-time models.

- **Period** — `Inf` specifies a nonperiodic input. For a periodic input, type the period of the input signal in your experiment.

  Note If your data is periodic, always include a whole number of periods for model estimation.

Channel Names

- **Input** — Enter `power`.

  Tip Naming channels helps you to identify data in plots. For multivariable input and output signals, you can specify the names of individual **Input** and **Output** channels, separated by commas.

- **Output** — Enter `temperature`.

Physical Units of Variables

- **Input** — Enter `W` for power units.

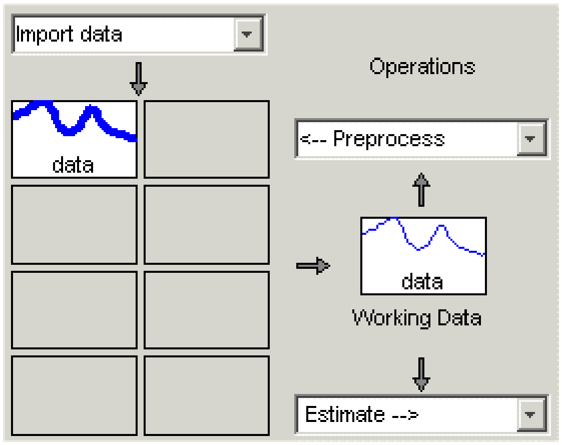
Tip When you have multiple inputs and outputs, enter a comma-separated list of **Input** and **Output** units corresponding to each channel.

- **Output** — Enter `^oC` for temperature units.

Notes

- Enter comments about the experiment or the data. For example, you might enter the experiment name, date, and a description of experimental conditions. When you estimate models from this data, these models inherit your data notes.

4 Click **Import** to add the icon named `data` to the System Identification Tool window.



5 Click **Close** to close the Import Data dialog box.

Plotting and Preprocessing Data

In this portion of the tutorial, you examine the data and prepare it for system identification. You learn how to:

- Plot the data.

- Subtract the mean values of the input and the output to remove offsets.
- Split the data into two parts. You use one part of the data for model estimation, and the other part of the data for model validation.

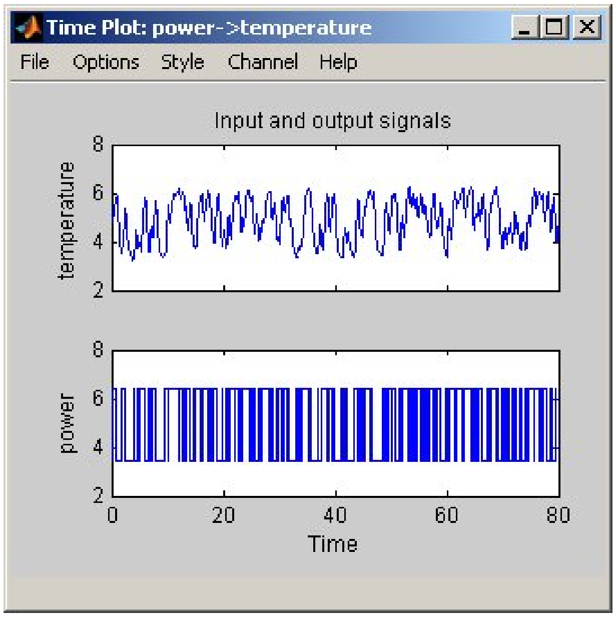
The reason you subtract the mean values from each signal is because, typically, you build linear models that describe the responses for deviations from a physical equilibrium. With steady-state data, it is reasonable to assume that the starting levels of the signals correspond to such an equilibrium. Thus, you can seek models around zero without modeling the absolute equilibrium levels in physical units.

You must have already imported data into the System Identification Tool, as described in "Importing Data Arrays into the System Identification Tool" on page 3-6.

Tip For information about other types of preprocessing, such as resampling and filtering the data, see the topics about plotting and preprocessing data in the *System Identification Toolbox User's Guide*.

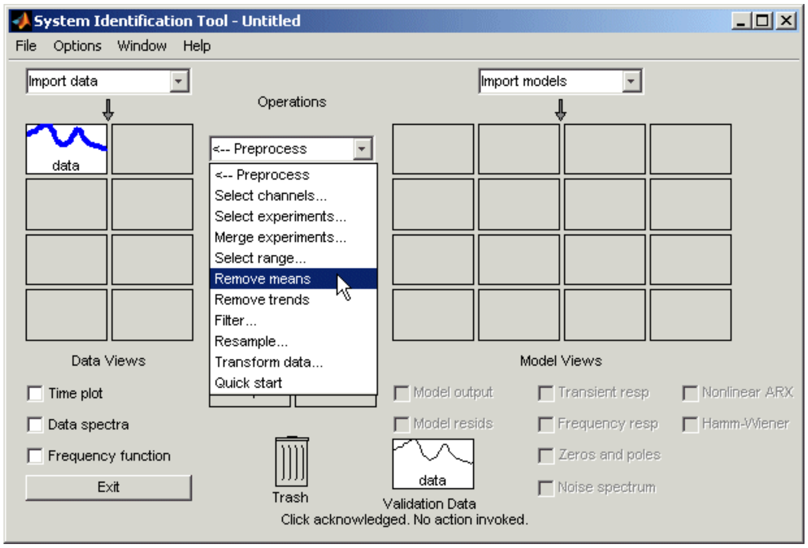
---

1 In the System Identification Tool window, select the **Time plot** check box to open the Time Plot.


Time Plot: power->temperature
File Options Style Channel Help
Input and output signals
temperature
power
Time
8
6
4
2
0
20
40
60
80

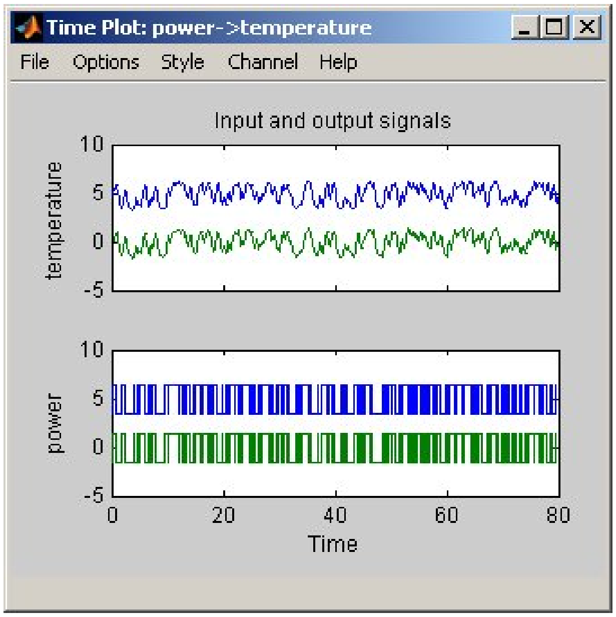
The top axes show the output data (temperature), and the bottom axes show the input data (power). Both the input and the output data have nonzero mean values.

2 In the System Identification Tool window, select **<--Preprocess > Remove means** to subtract the mean input value from the input data and the mean output value from the output data.



Click acknowledged. No action invoked.

This action adds a new data set to the System Identification Tool window with the default name `datad` (the suffix *d* means *detrend*), and updates the Time Plot window to display both the original and the detrended data.



The detrended data has a zero mean value.

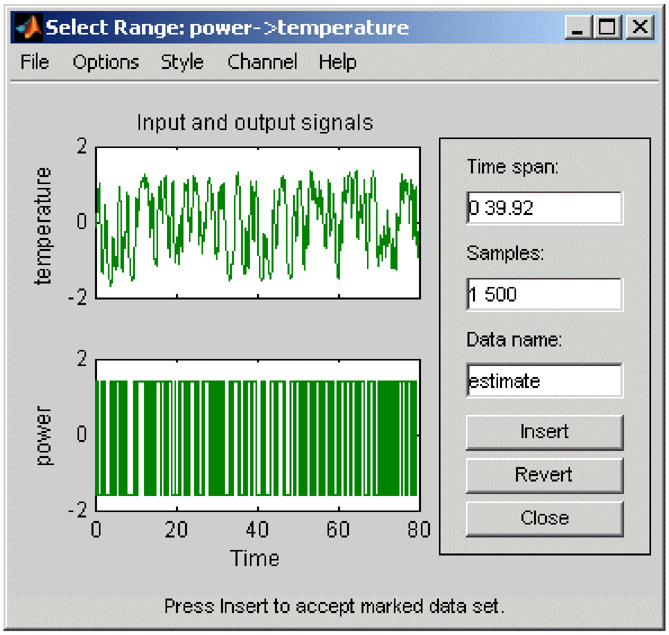
3 In the System Identification Tool window, drag the `datad` data set to the **Working Data** rectangle. This action specifies the detrended data to be used for estimating models.

4 Select **<--Preprocess > Select range** to open the Select Range window.

In this window, you can split the data into two parts and specify the first part for model estimation, and the second part for model validation, as described in the following steps.

5 In the Select Range window, change the **Samples** field to select the first 500 samples, as follows:

```
1 500
```



Time span: 0 39.92

Samples: 1 500

Data name: estimate

Press Insert to accept marked data set.

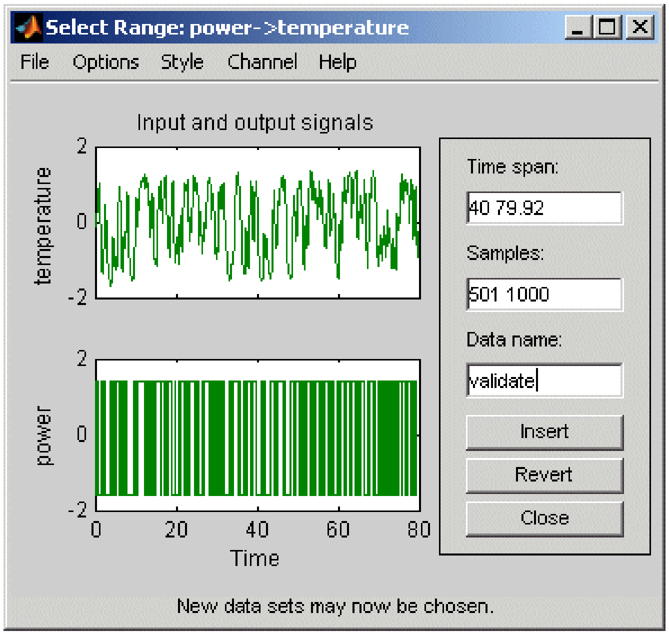
Tip You can also select data samples using the mouse by clicking and dragging a rectangular region on the plot. If you select samples on the input-channel axes, the corresponding region is also selected on the output-channel axes.

---

6 In the **Data name** field, type the name `estimate`, and click **Insert**. This action adds a new data set to the System Identification Tool window to be used for model estimation.

7 In the Select Range window, change the **Samples** field to select the last 500 samples, as follows:

```
501  1000
```



Time span: 40 79.92

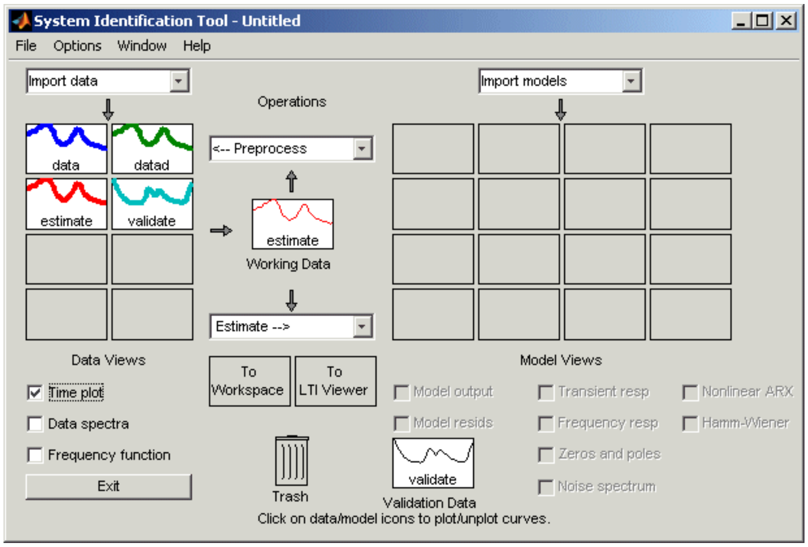
Samples: 501 1000

Data name: validate

New data sets may now be chosen.

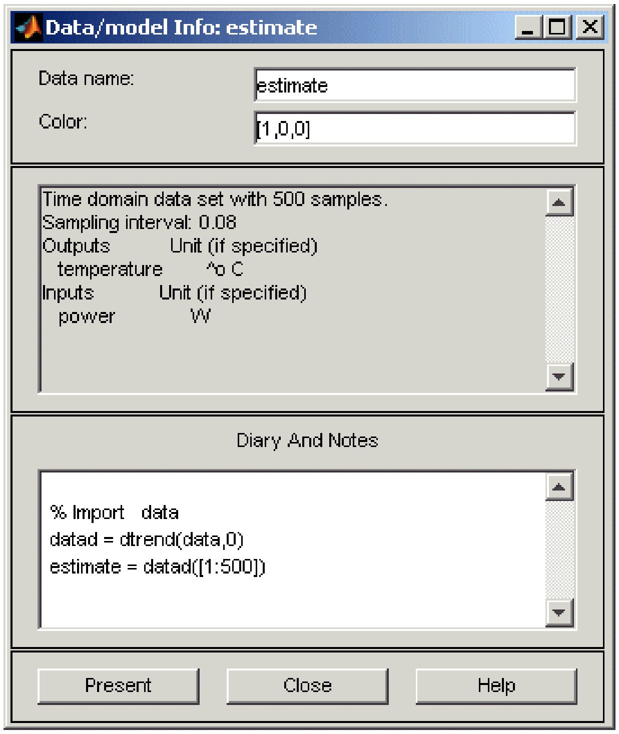
8 In the **Data name** field, type the name `validate`, and click **Insert**. This action adds a new data set to the System Identification Tool window to be used for model validation.

9 Drag and drop `estimate` to the **Working Data** rectangle, and drag and drop `validate` to the **Validation Data** rectangle so that the System Identification Tool window resembles the following figure.



Tip If you have multiple data sets available from different experiments, you can use one data set for estimation and another data set for validation. Thus, you need not split the data set you originally imported.

10 To get information about a data set, right-click its icon. For example, right-click the `estimate` data set to open the Data/model Info dialog box.



Data name: estimate

Color: [1,0,0]

```
Time domain data set with 500 samples.
Sampling interval: 0.08
Outputs          Unit (if specified)
  temperature       ^o C
Inputs           Unit (if specified)
  power            W
```

```
% Import   data
datad = dtrend(data,0)
estimate = datad([1:500])
```

In the Data/model Info dialog box, you can perform the following actions:

- Change the name of the data set in the **Data name** field.
- Change the color of the data icon by changing the RGB values (relative amounts of red, green, and blue). Each value is between `0` and `1`. For example, `[1,0,0]` indicates that only red is present, and no green and blue are mixed into the overall color.
- In the noneditable area, view the total number of samples, the sampling interval, and the output and input channel names and units.
- In the editable **Diary And Notes** area, view or edit the actions you performed on this data set. The actions are translated into commands equivalent to your GUI operations. For example, the `estimate` data set is a result of importing the data, detrending the mean values, and selecting the first 500 samples of the data:

  ```
  load dryer2
  % Import data
  datad = detrend(data,0)
  estimate = datad([1:500])
  ```

  For more information on these and other toolbox commands, see the reference page for each command.

Tip As an alternative preprocessing shortcut, you can select **Preprocess > Quick start** from the System Identification Tool window to simultaneously perform all of the data preprocessing steps in this tutorial.

For information about other types of preprocessing, such as resampling and filtering data, see the *System Identification Toolbox User's Guide*.

Saving the GUI Session

After you preprocess the data, as described in "Plotting and Preprocessing Data" on page 3-11, you may delete any data sets in the window that you do not need for estimation and validation, and save your session. You can open this session later and use it as a starting point for model estimation and validation without repeating these preparatory steps.

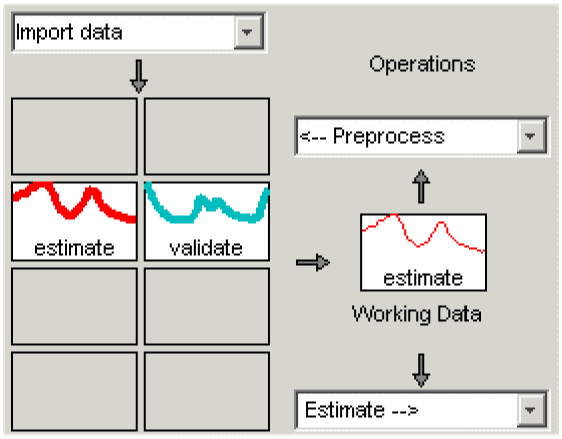
In the following procedure, you delete the original data set `data` and the detrended data set `datad`, rearrange the data icons in the System Identification Tool window, and save the session.

1. In the System Identification Tool window, drag and drop the `data` data set into the **Trash**.

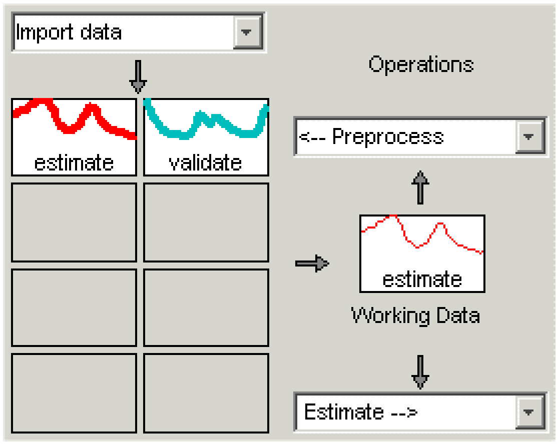
2. Drag and drop the `datad` data set into the **Trash**.

Note Moving items to the **Trash** does not delete them. To permanently delete items, select **Options > Empty trash** in the System Identification Tool window.

The following figure shows the System Identification Tool window after moving the items to the **Trash**.



3 Drag and drop the `estimate` and `validate` data sets to fill the empty rectangles, as shown in the following figure.



4 Select **File > Save session as** to open the Save Session dialog box, and browse to the directory where you want to save the session file.

5 In the **File name** field, type the name of the session `prep_data`, and click **Save**. The resulting file has a `.sid` extension.

Tip To open a saved session when starting the System Identification Tool, type the session name as an argument. For example:

```
ident('prep_data')
```

For more information about managing sessions, see the topics on working with the System Identification Tool GUI in the *System Identification Toolbox User's Guide*.

Estimating Preliminary Models

In this section...

Why Estimate Preliminary Models?

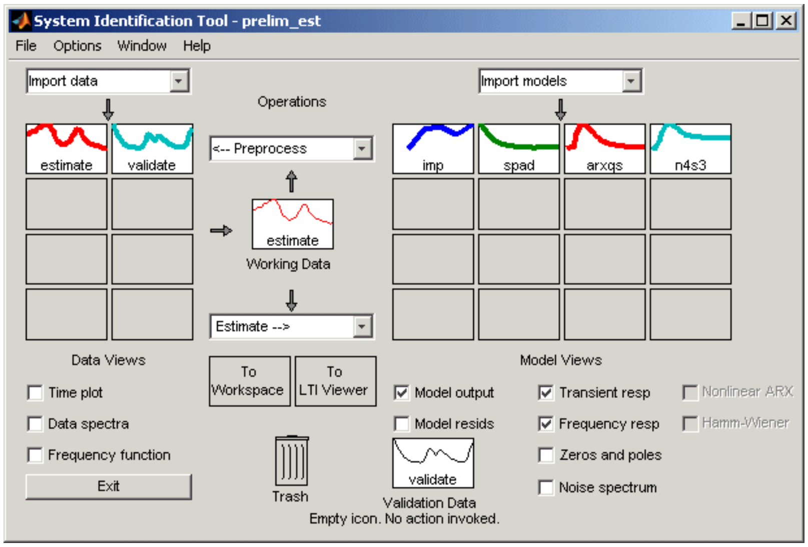
After preparing the data for estimation, as described in "Plotting and Preprocessing Data" on page 3-11, you can use the Quick Start feature of System Identification Toolbox to estimate and compare several types of models. You can use these models to assess whether linear modeling is sufficient. Preliminary models also help you gain insight into the possible delays and orders of the model that you can later refine.

Using Quick Start to Estimate Preliminary Models

To generate preliminary models, select **Estimate > Quick start** in the System Identification Tool window.

This action generates plots of impulse response, frequency-response, and the output of state-space and polynomial models. For more information about these plots, see "Validating Preliminary Models" on page 3-24. Close the model plots after you examine them.

For a description of the generated models, see "Types of Models Generated by Quick Start" on page 3-28.



Validating Preliminary Models

Estimating models using Quick Start generates the following three plots of the preliminary models you created in "Using Quick Start to Estimate Preliminary Models" on page 3-24:

- Step-response plot
- Frequency-response plot

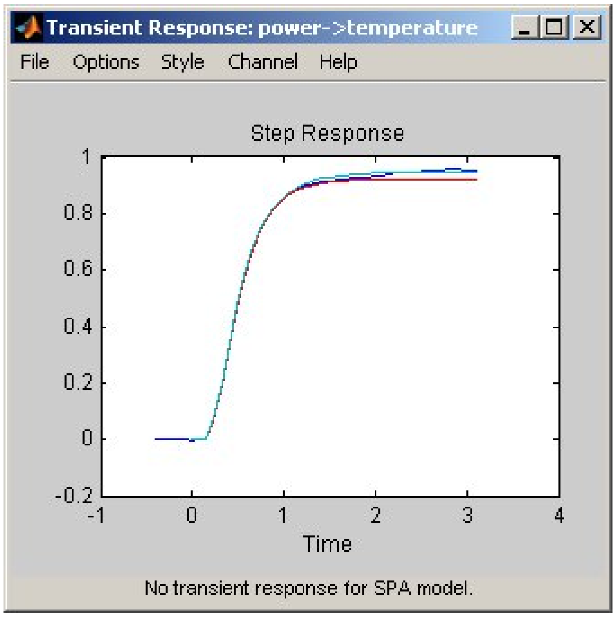
- Model-output plot

You can analyze these plots to determine the quality of the model. Close the model plots after you examine them.

Step-Response Plot

The following step-response plot shows agreement for the different models.

Tip If you closed the plot window, select the **Transient resp** check box to reopen this window.


Transient Response: power->temperature
File
Options
Style
Channel
Help
Step Response
1
0.8
0.6
0.4
0.2
0
-0.2
-1
0
1
2
3
4
Time
No transient response for SPA model.

Step Response for imp, arxqs, and n4s3

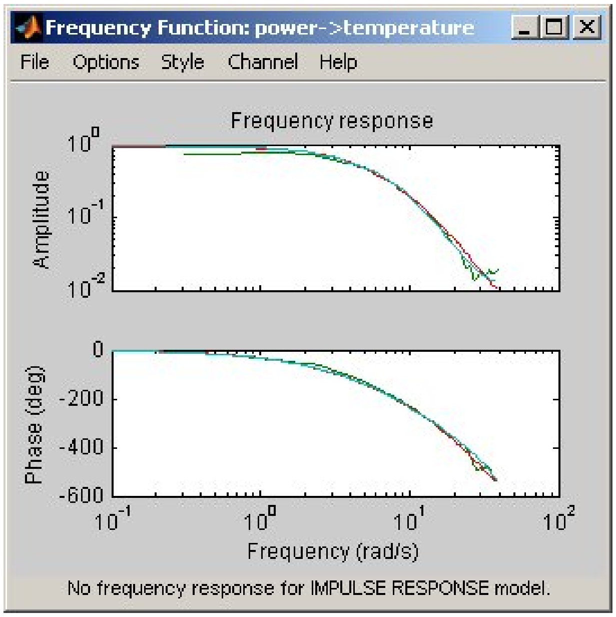
Note The step-response plot does not include the frequency-response model, `spad`, estimated using spectral analysis.

Tip You can use the step-response plot to estimate the dead time of linear systems. For example, the previous step-response plot shows a time delay of about 0.25 s before the system responds to the input. This response delay, or *dead time*, is approximately equal to about three samples because the sampling interval is `0.08` s for this data set.

Frequency-Response Plot

The following frequency-response plot shows agreement for the different models.

Tip If you closed this plot window, select the **Frequency resp** check box to reopen this window.



No frequency response for IMPULSE RESPONSE model.

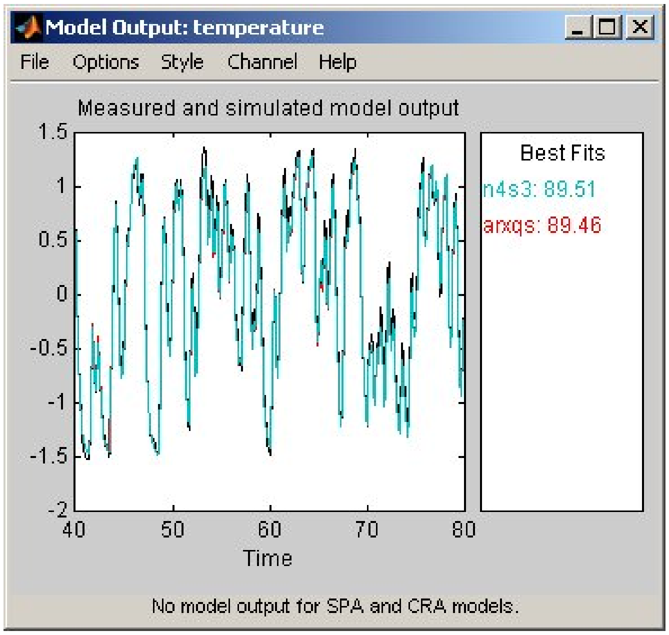
Frequency Response for Models spad, arxqs, and n4s3

Note The frequency-response plot does not include the impulse-response model, `imp`, which is estimated using correlation analysis.

Model-Output Plot

The Model Output window displays this model output together with the measured output in the validation data.

Tip If you closed the Model Output window, select the **Model output** check box to reopen this window.


Model Output: temperature
File
Options
Style
Channel
Help
Measured and simulated model output
Best Fits
n4s3: 89.51
arxqs: 89.46
1.5
1
0.5
0
-0.5
-1
-1.5
-2
40
50
60
70
80
Time
No model output for SPA and CRA models.

Measured Output and Model Output for Models arxqs and n4s3

The model-output plot shows the model response to the input in the validation data. The fit values for each model are summarized in the **Best Fits** area of the Model Output window. The models in the **Best Fits** list are ordered from best at the top to worst at the bottom. The fit between the two curves is computed such that 100 means a perfect fit, and 0 means that the model output has the same fit to the measured output as the mean of the measured output.

In this example, the output of the models matches the validation data output, which indicates that the models seem to capture the main system dynamics and that linear modeling is sufficient.

Tip To compare predicted model output instead of simulated output, select this option from the **Options** menu in the Model Output window.

Types of Models Generated by Quick Start

Quick Start estimates the following four types of models and adds the following to the System Identification Tool window with default names:

- `imp` — Step response by correlation analysis using `impulse`.

  This model is nonparametric (not expressed in terms of parameters).

- `spad` — Spectral analysis estimate of the frequency function using `spa`. The frequency function is the Fourier transform of the impulse response of a linear system.

  This model is nonparametric and computes the response for each frequency value. By default, the model is evaluated at 128 frequency values, ranging from 0 to the Nyquist frequency.

- `arxqs` — Fourth-order autregressive (ARX) model calculated using `arx`.

  This model is parametric and has the following structure:

  $$\begin{aligned} y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-n_a) &= \\ \mathrm{b}_1 u(t-n_k) + \ldots + b_{nb} u(t-n_k-n_b+1) + e(t) \end{aligned}$$

  y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, n_a is the number of poles, n_b is the number of *b* parameters (equal to the number of zeros plus 1), n_k is the number of samples before the input affects output of the system (called *dead time*), and *e(t)* is the white-noise disturbance.

  System Identification Toolbox estimates the parameters $a_1 \ldots a_n$ and $b_1 \ldots b_n$ using the input and output data from the estimation data set.

  In `arxqs`, n_a=n_b=4, and n_k is estimated from the step response model `imp`.

- `n4s3` — State-space model calculated using `n4sid`. The algorithm automatically selects the model order (in this case, 3).

  This model is parametric and has the following structure:

  $$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned}$$

  y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, *x* is the state vector, and *e(t)* is the white-noise disturbance. System Identification Toolbox estimates the state-space matrices *A*, *B*, *C*, *D*, and *K*.

Estimating Accurate Models

In this section...

Strategy for Getting Accurate Models

Because the simple models in "Estimating Preliminary Models" on page 3-23 showed that a linear model sufficiently represents the dynamics of the Feedback Process Trainer, it is worthwhile to improve the model accuracy.

In this portion of the tutorial, you get accurate parametric models by performing the following activities:

1. Identifying initial model orders and delays from your data using a simple, autoregressive model structure (ARX).
2. Exploring more complex model structures with orders and delays close to the initial values you found .

The resulting models are discrete-time models.

Tip You can convert a discrete-time model to a continuous-time model using the `d2c` command. For more information, see the corresponding reference page.

Estimating a Range of Model Orders

What are the reasonable model orders for your system? You can estimate simple autoregressive (ARX) models to get a range of orders and delays and compare the performance of these models. Use the orders and delays that results in the best model fit as an initial guess for more accurate modeling.

About ARX Models

For a single-input/single-output system (SISO), the ARX model structure is:

$$\begin{aligned} &y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-n_a) = \\ &\qquad \mathrm{b}_1 u(t-n_k) + \ldots + b_{nb} u(t-n_k-n_b+1) + e(t) \end{aligned}$$

y(t) represents the output at time t, *u(t)* represents the input at time t, n_a is the number of poles, n_b is the number of zeros plus 1, n_k is the number of samples before the input affects the system output, and *e(t)* is the white-noise disturbance.

You must specify the model orders to estimate ARX models.

System Identification Toolbox estimates the parameters $a_1 \ldots a_n$ and $b_1 \ldots b_n$ using the data and the model orders you specify.

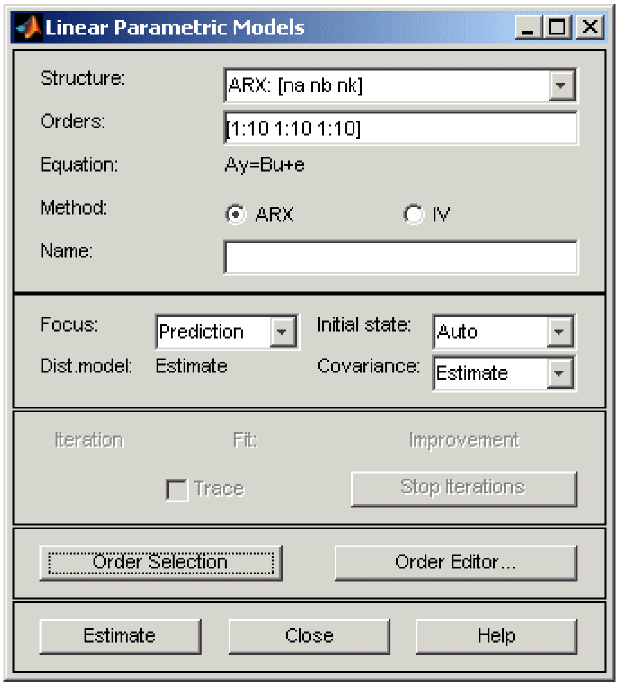
How to Estimate Model Orders

1 In the System Identification Tool window, select **Estimate > Linear parametric models** to open the Linear Parametric Models dialog box.

  The ARX model is already selected by default in the **Structure** list.

2 Edit the **Orders** field to specify that System Identification Toolbox try all combinations of poles, zeros, delays, where each value is between 1 and 10:

```
[1:10 1:10 1:10]
```



Linear Parametric Models

Structure: ARX: [na nb nk]

Orders: [1:10 1:10 1:10]

Equation: Ay=Bu+e

Method: [✓ ARX] IV

Name:

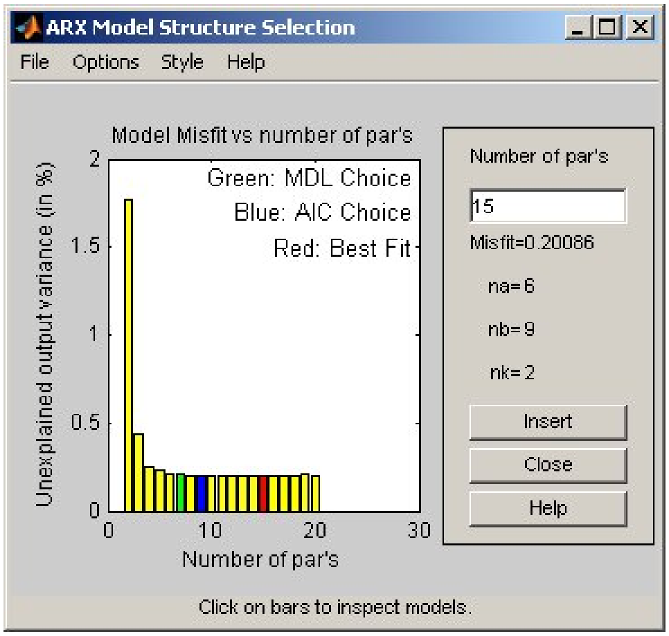
Focus: Prediction

Initial state: Auto

Dist.model: Estimate

Covariance: Estimate

3 Click **Estimate** to open the ARX Model Structure Selection window, which displays the model performance for each combination of model parameters.



You use this plot to select the best-fit model. The horizontal axis is the total number of parameters:

$$\text{Number of parameters} = n_a + n_b$$

The vertical axis, called **Unexplained output variance (in %)**, is the portion of the output not explained by the model—the ARX model prediction error for a specific number of parameters. The *prediction error* is the sum of the squares of the differences between the validation data output and the model output.

Three rectangles are highlighted on the plot in green, blue, and red. Each color indicates a type of best-fit criterion, as follows:

- Red — Best fit minimizes the sum of the squares of the difference between the validation data output and the model output. This rectangle indicates the overall best fit.
- Green — Best fit minimizes Rissanen MDL criterion.
- Blue — Best fit minimizes Akaike AIC criterion.

In this tutorial, the **Unexplained output variance (in %)** value remains approximately constant for the combined number of parameters from 4 to 20. Such constancy indicates that model performance does not improve at higher orders. Thus, low-order models might fit the data equally well.

Note When you use the same data set for estimation and validation, use the MDL and AIC criteria to select model orders. These criteria compensate for overfitting that results from using too many parameters.

---

4 In the ARX Model Structure Selection window, select the red bar (corresponding to `15` on the horizontal axis), and click **Insert**. This selection inserts n_a=6, n_b=9, and n_k=2 into the Linear Parametric Models dialog box and performs the estimation.

System Identification Toolbox adds the model `arx692` to the System Identification Tool window and updates the plots to include the response of the model.

Note The default name of the parametric model contains the model type and the number of poles, zeros, and delays. For example, `arx692` is an ARX model with n_a=6, n_b=9, and a delay of two samples.

---

5 In the ARX Model Structure Selection window, select the bar corresponding to `4` on the horizontal axis (the lowest order that still gives a good fit), and click **Insert**.

- This selection inserts n_a=2, n_b=2, and n_k=3 (a delay of three samples) into the Linear Parametric Models dialog box and performs the estimation.
- The model `arx223` is added to the System Identification Tool window and the plots are updated to include its response and output.

6 Click **Close** to close the ARX Model Structure Selection window.

Estimating State-Space and ARMAX Models

By estimating ARX models with different combinations of orders, as described in "Estimating a Range of Model Orders" on page 3-30, you identified the number of poles, zeros, and delays that provide a good starting point for systematically exploring different models.

The overall best fit for this system corresponds to a model with six poles, nine zeros, and a delay of two samples. It also showed that a low-order model with n_a=2 (two poles), n_b=2 (one zero), and n_k=3 also provides a good fit.

About State-Space Models

The general state-space model structure is:

$$x(t+1) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, *x(t)* is the state values at time *t*, and *e(t)* is the white-noise disturbance.

You must specify a single integer as the model order to estimate a state-space model. By default, the delay equals `1`.

System Identification Toolbox estimates the state-space matrices *A*, *B*, *C*, *D*, and *K* using the model order and the data you specify.

The state-space model structure is a good choice for quick estimation because it contains only two parameters: `n` is the number of poles (the size of the *A* matrix) and `nk` is the delay.

About ARMAX Models

For a single-input/single-output system (SISO), the ARMAX model structure is:

$$\begin{aligned}
&y(t)+a_1y(t-1)+\ldots+a_{na}y(t-n_a)= \\
&\qquad \mathrm{b}_1u(t-n_k)+\ldots+b_{nb}u(t-n_k-n_b+1)+ \\
&\qquad e(t)+c_1e(t-1)+\ldots+c_{nc}e(t-n_c)
\end{aligned}$$

y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, n_a is the number of poles for the dynamic model, n_b is the number of zeros plus 1, n_c is the number of poles for the disturbance model, n_k is the dead time (in terms of the number of samples) before the input affects output of the system, and *e(t)* is the white-noise disturbance.

Note The ARMAX model is more flexible than the ARX model because the ARMAX structure contains an extra polynomial to model the additive disturbance.

You must specify the model orders to estimate ARMAX models.

System Identification Toolbox estimates the parameters $a_1\ldots a_n$, $b_1\ldots b_n$, and $c_1\ldots c_n$ using the data and the model orders you specify.

How to Estimate State-Space and ARMAX Models

To explore the state-space and ARMAX model structures:

1. In the System Identification Tool window, select **Estimate > Linear parametric models** to open the Linear Parametric Models dialog box.
2. From the **Structure** list, select `State Space:  n [nk]`.
3. In the **Orders** field, type `6` to create a sixth-order state-space model.

   This choice is based on the fact that the best-fit ARX model has six poles.

4 Click **Estimate** to add a state-space model called `n4s6` to the System Identification Tool window.

5 From the **Structure** list, select `ARMAX: [na nb nc nk]` to estimate an ARMAX model.

6 In the **Orders** field, set the orders *na*, *nb*, *nc*, and *nk* to the following values:

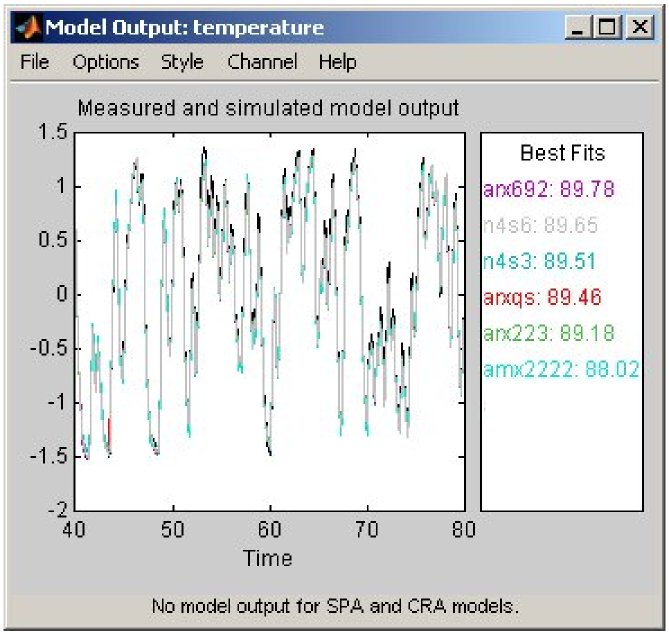
```
2 2 2 2
```

The model name in the **Name** field is `amx2222`, by default.

7 Click **Estimate** to add the ARMAX model to the System Identification Tool window.

Tip If you closed the Model Output window, you can regenerate it by selecting the **Model output** check box in the System Identification Tool window. If the new model does not appear on the plot, click the model icon in the System Identification Tool window to make the model active.

The fit for `amx2222` is about 1% lower than the other models.


Model Output: temperature
File Options Style Channel Help
Measured and simulated model output
1.5
1
0.5
0
-0.5
-1
-1.5
-2
40
50
60
70
80
Time

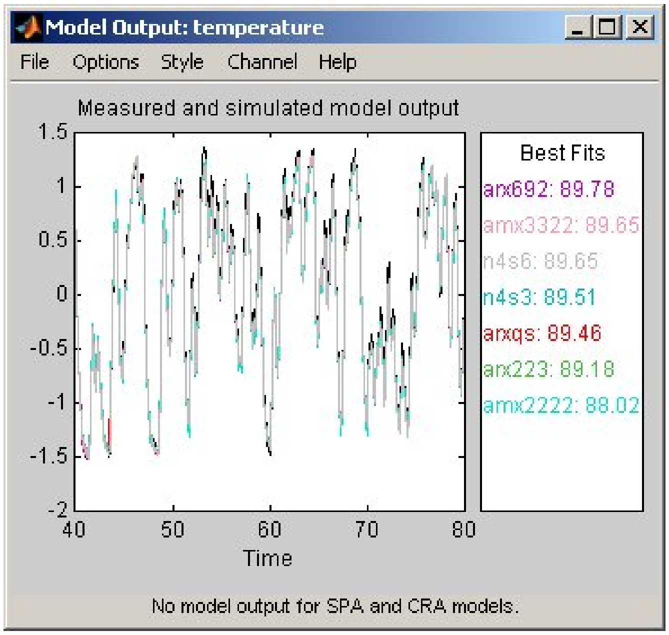
Best Fits
arx692: 89.78
n4s6: 89.65
n4s3: 89.51
arxqs: 89.46
arx223: 89.18
amx2222: 88.02

No model output for SPA and CRA models.

Note The **Best Fits** area in the Model Output window sorts models such that models with the best-fit model appear at the top.

---

8 Repeat steps 6 and 7 using higher **Orders** `3 3 2 2`. These orders result in a model that fits the data almost as well as the higher order ARX model `arx692`.



Learn More

To learn more about estimating state-space models, see the corresponding section in the *System Identification Toolbox User's Guide*.

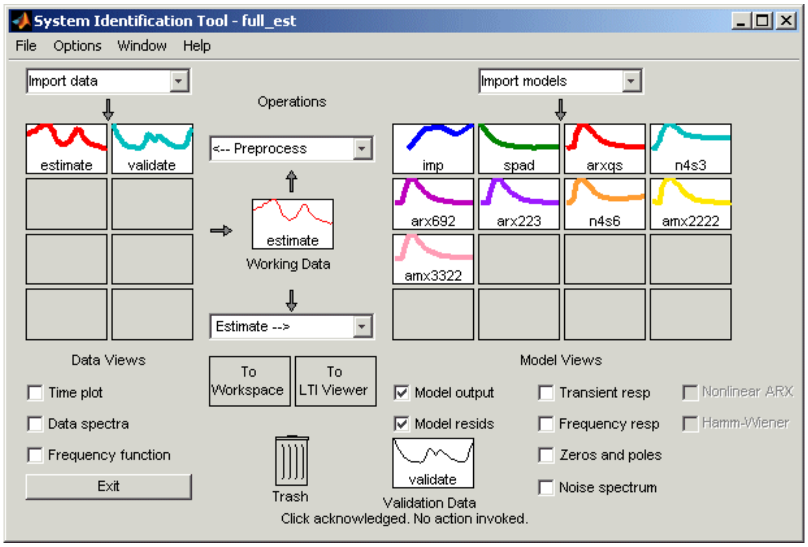
To learn more about estimating polynomial models, such as ARMAX, see the corresponding section in the *System Identification Toolbox User's Guide*.

Choosing the Best Model

- "Summary of Models" on page 3-40
- "Examining the Model Output" on page 3-40
- "Examining Model Residuals" on page 3-43

Summary of Models

The following figure shows the System Identification Tool window, which includes all of the models you estimated so far.



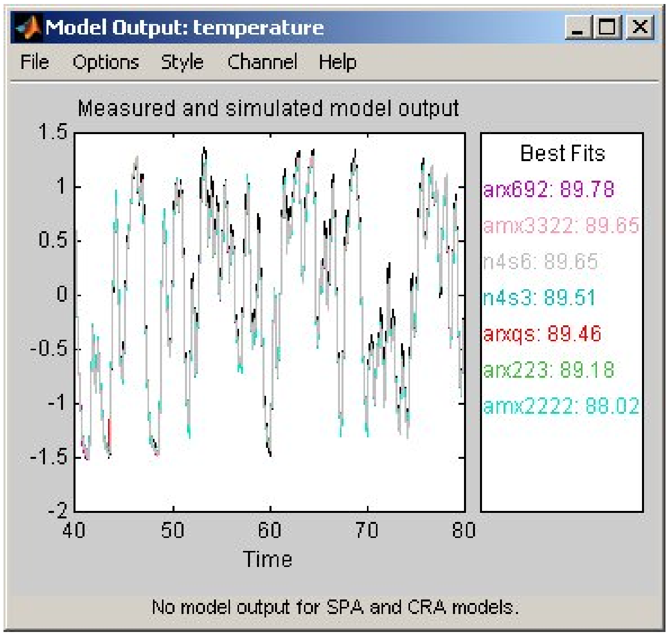
Examining the Model Output

A good model is the simplest model that best explains the dynamics and successfully simulates or predicts the output for different inputs.

The Model Output window should be already open. It is automatically updated with the new models. Examine the model-output plot to see how well the model output matches the measured output in the validation data set.

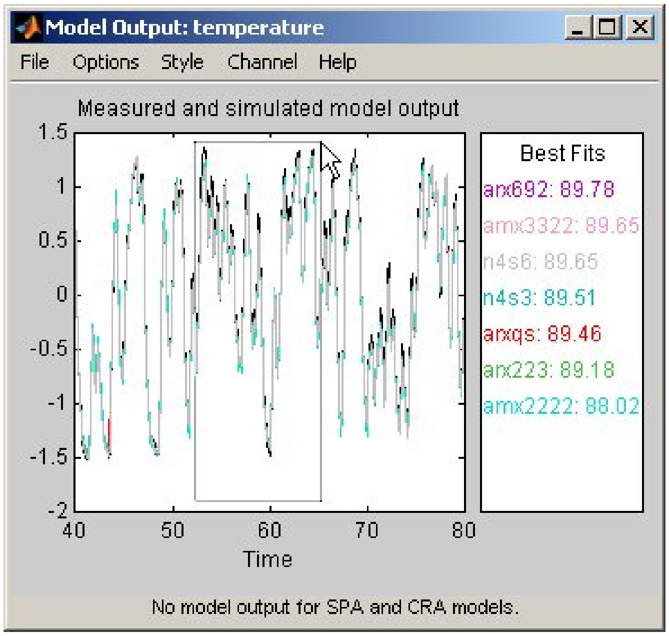
Tip If you closed the Model Output plot, you can regenerate it by selecting the **Model output** check box in the System Identification Tool window. If the new model does not appear on the plot, click the model icon in the System Identification Tool window to include this model on plots.

Models are listed by name in the **Best Fits** area of the Model Output plot. The highest-order model you created, `arx692`, fits the data as well as the simpler model `amx3322`.



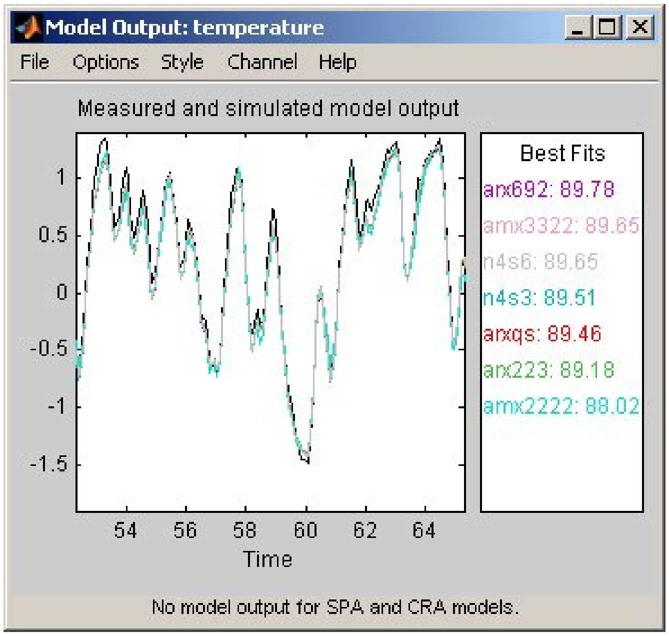
Tip To validate your models using a different data set, you can drag and drop this data set into the **Validation Data** rectangle in the System Identification Tool window. This action automatically updates any open model views. If you transform validation data into the frequency domain, the model-output plot updates to show the model comparison in the frequency domain.

To get a closer look at how well these models fit the data, magnify a portion of the plot by clicking and dragging a rectangle around the region of interest, as shown in the following figure.



No model output for SPA and CRA models.

Releasing the mouse magnifies this region and shows that all models seem to agree with the validation data.


Model Output: temperature
File Options Style Channel Help
Measured and simulated model output
1
0.5
0
-0.5
-1
-1.5
54 56 58 60 62 64
Time

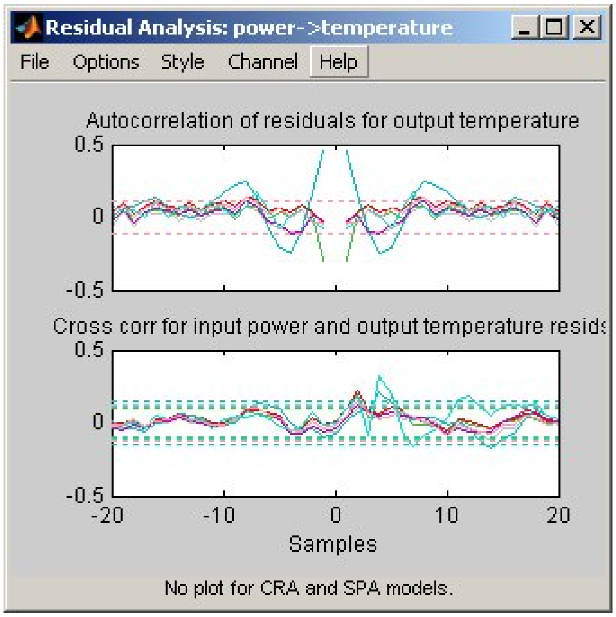
Best Fits
arx692: 89.78
amx3322: 89.65
n4s6: 89.65
n4s3: 89.51
arxqs: 89.46
arx223: 89.18
amx2222: 88.02

No model output for SPA and CRA models.

Examining Model Residuals

In addition to comparing model output to measured output, you can validate a model by checking the behavior of its residuals.

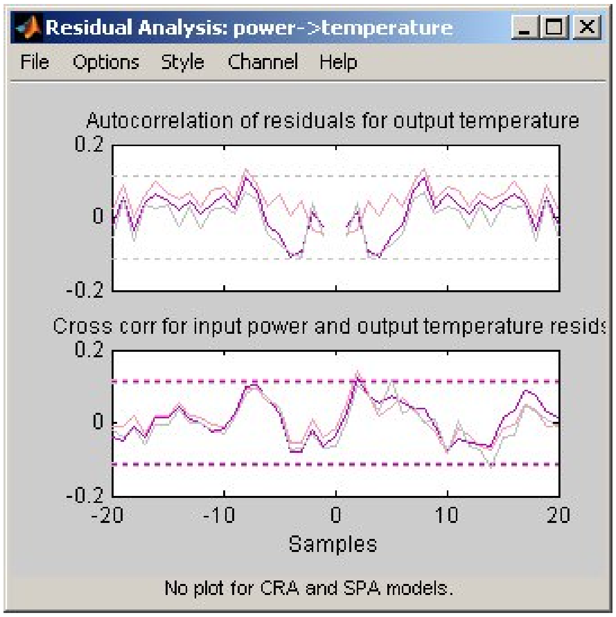
To generate a residual analysis plot, select the **Model resids** check box in the System Identification Tool window.


Residual Analysis: power->temperature
File Options Style Channel Help
Autocorrelation of residuals for output temperature
0.5
0
-0.5
Cross corr for input power and output temperature resids
0.5
0
-0.5
-20 -10 0 10 20
Samples
No plot for CRA and SPA models.

The top axes show the autocorrelation of residuals for the temperature output (whiteness test). The horizontal scale is the number of lags, or the difference between the time steps that are correlated. The horizontal dashed lines on the plot represent the model confidence interval. Any fluctuations within the confidence interval are considered to be insignificant. Two of the models, `n4s3` and `arx223`, produce residuals that enter outside the confidence interval. A good model should have a residual autocorrelation function within the confidence interval, indicating that the residuals are uncorrelated.

The bottom axes show the cross-correlation of the residuals with the input. A good model should have residuals uncorrelated with past inputs (independence test). Evidence of correlation indicates that the model does not describe how a portion of the output relates to the corresponding input. For example, when there is a peak outside the confidence interval for lag *k*, this means that the contribution to the output *y(t)* that originates from the input *u(t-k)* is not properly described by the model. The models `arxqs` and `amx2222` extend beyond the confidence interval and do not perform as well as the other models.

Click the model icons `n4s3`, `arx223`, `arxqs`, and `amx2222` in the System Identification Tool window to remove them from the Residual Analysis plot. The Residual Analysis plot now includes only the three models that pass the residual tests: `arx692`, `n4s6`, and `amx3322`.


Residual Analysis: power->temperature
File Options Style Channel Help
Autocorrelation of residuals for output temperature
0.2
0
-0.2
Cross corr for input power and output temperature resids
0.2
0
-0.2
-20
-10
0
10
20
Samples

No plot for CRA and SPA models.

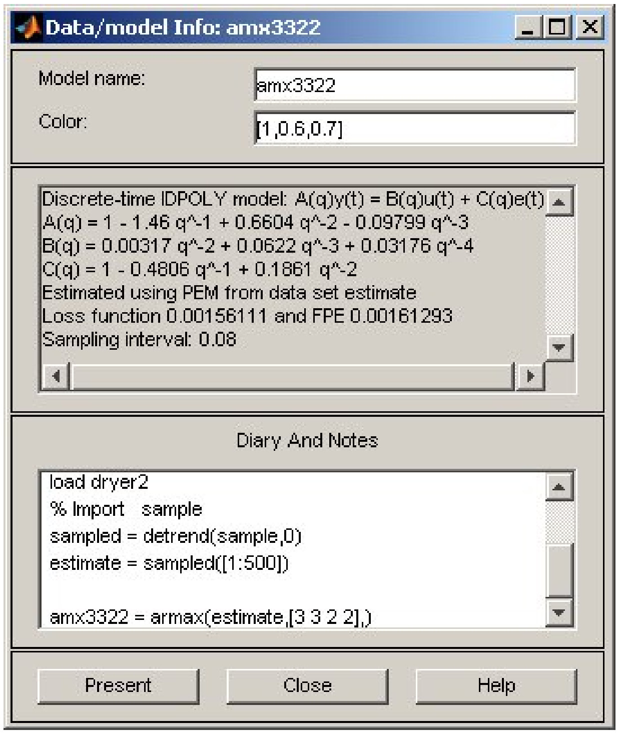
The plots for these models fall within the confidence intervals. Therefore, it is reasonable to pick the simpler, low-order model `amx3322` as the final choice. The `amx3322` output agrees well with the measured output.

Viewing Model Parameters

In this section...

Viewing Model Parameter Values

You can view the numerical parameter values of the model amx3322 by right-clicking the model icon in the System Identification Tool window. The Data/model Info dialog box opens.



Data/model Info: amx3322

Model name: amx3322

Color: [1,0.6,0.7]

```
Discrete-time IDPOLY model: A(q)y(t) = B(q)u(t) + C(q)e(t)
A(q) = 1 - 1.46 q^-1 + 0.6604 q^-2 - 0.09799 q^-3
B(q) = 0.00317 q^-2 + 0.0622 q^-3 + 0.03176 q^-4
C(q) = 1 - 0.4806 q^-1 + 0.1861 q^-2
Estimated using PEM from data set estimate
Loss function 0.00156111 and FPE 0.00161293
Sampling interval: 0.08
```

Diary And Notes

```
load dryer2
% Import   sample
sampled = detrend(sample,0)
estimate = sampled([1:500])

amx3322 = armax(estimate,[3 3 2 2],)
```

The noneditable area of the Data/model Info dialog box lists the following parameter values:

```
A(q) = 1 - 1.46q^-1 + 0.6604q^-2 - 0.09799q^-3
B(q) = 0.00317q^-2 + 0.0622q^-3 + 0.03176q^-4
C(q) = 1 - 0.4806q^-1 + 0.1861q^-2
```

These parameter values correspond to the following difference equation for your system:

$$\begin{aligned} & y(t) - 1.46y(t-1) + 0.6604y(t-2) - 0.09799y(t-3) = \\ & \quad 0.00317u(t-2) + 0.622u(t-3) + 0.03176u(t-4) + \\ & \quad e(t) - 0.4806e(t-1) + 0.1861e(t-2) \end{aligned}$$

Note The coefficient of *u(t-2)* is not significantly different from zero. This lack of difference explains why delay values of both 2 and 3 give good results.

Parameter values appear in the following format:

$$\begin{aligned} & A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \\ & B(q) = b_1 q^{-1} + \ldots + b_{nb} q^{-nb} \\ & C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc} \end{aligned}$$

The parameters appear in the ARMAX model structure, as follows:

$$A(q)y(t) = B(q)u(t) + C(q)e(t)$$

which corresponds to this general difference equation:

$$\begin{aligned} y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-n_a) = \\ \mathrm{b}_1 u(t-n_k) + \ldots + b_{nb} u(t-n_k-n_b+1) + \\ e(t) + c_1 e(t-1) + \ldots + c_{nc} e(t-n_c) \end{aligned}$$

y(t) represents the output at time t, *u(t)* represents the input at time t, n_a is the number of poles for the dynamic model, n_b is the number of zeros plus 1, n_c is the number of poles for the disturbance model, n_k is the dead time (in terms of the number of samples) before the input affects output of the system, and *e(t)* is the white-noise disturbance.

Viewing Parameter Uncertainties

To view parameter uncertainties, click **Present** in the Data/model Info dialog box, and view the model information in the MATLAB Command Window.

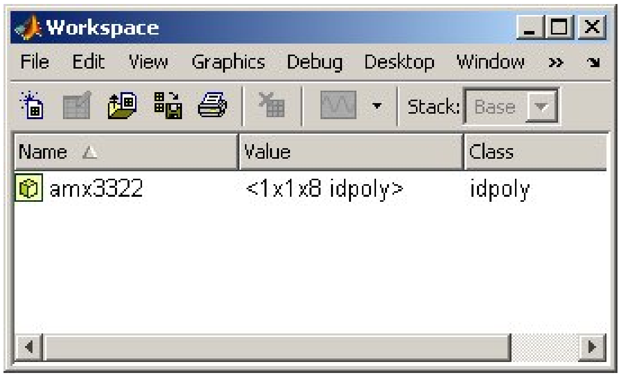
```
A(q) = 1 - 1.46(+-0.06003)q^-1
         + 0.6604(+-0.08906)q^-2
         - 0.09799(+-0.03519)q^-3
B(q) = 0.00317(+-0.001646)q^-2
         + 0.0622(+-0.002425)q^-3
         + 0.03176(+-0.005629)q^-4
C(q) = 1 - 0.4806(+-0.07558)q^-1
           + 0.1861(+-0.05597)q^-2
```

The 1-standard-deviation uncertainty for each set of model parameters is in parentheses next to each parameter value.

Exporting the Model to the MATLAB Workspace Browser

The models you create in the System Identification Tool GUI are not automatically available in the MATLAB Workspace browser. To make a model available to other toolboxes, Simulink, and System Identification Toolbox commands, you must export your model from the System Identification Tool to the MATLAB Workspace browser. For example, if the model is a plant that requires a controller, you can import the model from the MATLAB Workspace browser into Control System Toolbox.

To export the `amx3322` model, drag it to the **To Workspace** rectangle in the System Identification Tool window. The model appears in the MATLAB Workspace browser.



Name	Value	Class
amx3322	<1x1x8 idpoly>	idpoly

Note This model is an object that belongs to the `idpoly` class. To learn more about this model object, see the corresponding reference page.

After the model is in the MATLAB Workspace browser, you can perform other operations on the model. For example, if you have Control System Toolbox installed, you might transform the model to a state-space LTI object using:

```
ss_model=ss(amx3322)
```

You can extract the dynamic model and ignore the noise model using the following command:

```
ss_model=ss_model('m')
```

Exporting the Model to the LTI Viewer

If you have Control System Toolbox installed on your computer, the **To LTI Viewer** rectangle appears in the System Identification Tool window.

The LTI Viewer is a graphical user interface for viewing and manipulating the response plots of linear models. It displays the following plots:

- Step- and impulse-response
- Bode, Nyquist, and Nichols
- Frequency-response singular values
- Pole/zero
- Response to general input signals
- Unforced response starting from given initial states (only for state-space models)

To plot a model in the LTI Viewer, drag and drop the model icon to the **To LTI Viewer** rectangle in the System Identification Tool window.

For more information about working with plots in the LTI Viewer, see the Control System Toolbox documentation.

Tutorial: Estimating Process Models Using the GUI

Viewing Process Model Parameters (p. 4-27)	How to view estimated model parameters and the history of operations on the model and the corresponding data.
Exporting the Model to the MATLAB Workspace Browser (p. 4-30)	How to make the model available to operations in the MATLAB Command Window for further processing with this toolbox or other MathWorks products.
Using Simulink with System Identification Toolbox (p. 4-31)	How to create and simulate a System Identification Toolbox model in Simulink.

About This Tutorial

In this section...

Objectives

Estimate and validate simple, continuous-time models from single-input/single-output (SISO) data to find the one that best represents your system dynamics.

After completing this tutorial, you will be able to accomplish the following tasks using the System Identification Tool GUI:

- Import data objects from the MATLAB Workspace browser into the GUI.
- Plot and preprocess the data.
- Estimate and validate low-order, continuous-time models from the data.
- Export models to the MATLAB Workspace browser.
- Export the model to the LTI Viewer for linear analysis (requires Control System Toolbox).

Sample Data

The sample data you use in this tutorial is in `proc_data.mat`, which contains 200 samples of simulated single-input/single-output SISO) time-domain data. The input is a random binary signal that oscillates between -1 and +1. White noise is added to the input with a standard deviation of 0.2, which results in a signal-to-noise ratio of about 20 dB. This data is simulated using a second-order system with underdamped modes (complex poles) and a peak response at 1 rad/s:

$$G(s) = \frac{1}{1 + 0.2s + s^2} e^{-2s}$$

The sampling interval of the simulation is 1 second.

Note This tutorial uses time-domain data to demonstrate how you can estimate linear models. However, the same workflow applies to frequency-domain data.

What Is a Continuous-Time Process Model?

Continuous-time process models are simple transfer functions that describe the system dynamics using static gain, time delay before the system output responds to the input, and characteristic time constants associated with poles and zeros. Such models are popular the process industry and are often used for tuning PID controllers, for example. The parameters of process models have physical significance.

You use the System Identification Tool to specify different process-model structures by varying the number of poles, adding an integrator, or adding or removing a time delay or a zero. The highest model order you can specify in this toolbox is three, and the poles can be real or complex (underdamped modes).

In general, a linear system is characterized by a transfer function *G*, which is an operator that takes the input *u* to the output *y*:

$$y = Gu$$

For a continuous-time system, *G* relates the Laplace transforms of the input *U(s)* and the output *Y(s)*:

$$Y(s) = G(s)U(s)$$

In this tutorial, you estimate *G* using different process-model structures.

For example, the following model structure is a first-order, continuous-time process model, where *K* is the static gain, T_{p1} is a time constant, and T_d is the input-to-output delay:

$$G(s) = \frac{K}{1 + sT_{p1}} e^{-sT_d}$$

Preparing Data

In this section...

Loading Data into the MATLAB Workspace Browser

Load sample data in `proc_data.mat` by typing the following command at the MATLAB prompt:

```
load proc_data
```

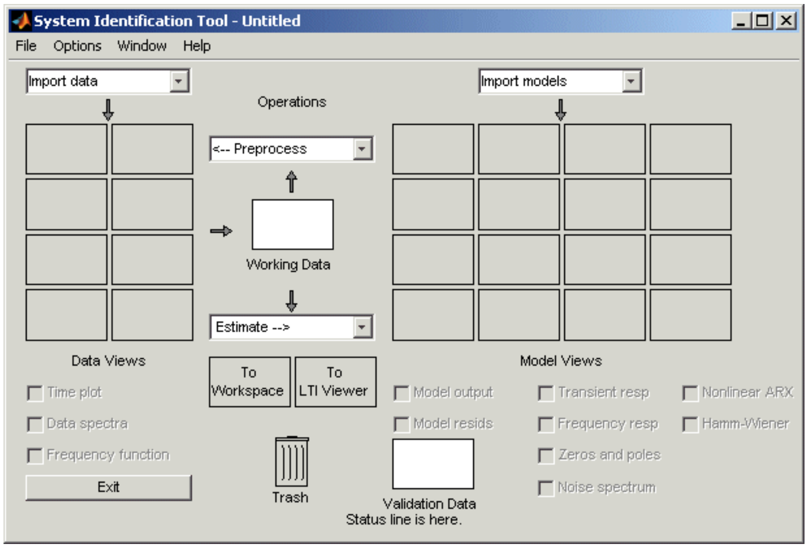
This command loads the data into the MATLAB Workspace browser as the data object `z`. For more information about `iddata` objects, see the corresponding reference pages.

Opening the System Identification Tool GUI

To open the System Identification Tool GUI, type the following command at the MATLAB prompt:

```
ident
```

The default session name, `Untitled`, appears in the title bar.

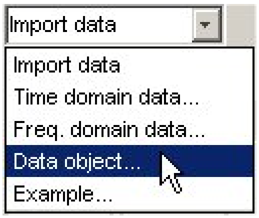


Importing Data Objects into the System Identification Tool

You can import the data objects into the GUI from the MATLAB Workspace browser.

You must have already opened the GUI, as described in "Opening the System Identification Tool GUI" on page 4-6.

1 In the System Identification Tool window, select **Import data > Data object**. This action opens the Import Data dialog box.

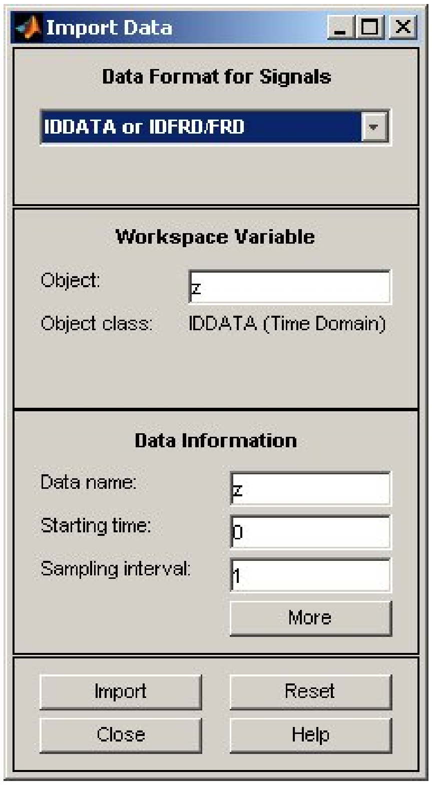


2 Specify the following options:

- **Object** — Enter `z` as the name of the MATLAB variable that is the time-domain data object. Press **Enter**.
- **Data name** — Use the default name `z`, which is the same as the name of the data object you are importing. This name labels the data in the System Identification Tool window after the import operation is completed.
- **Starting time** — Enter `0` as the starting time. This value designates the starting value of the time axis on time plots.
- **Sampling interval** — Enter `1` as the time between successive samples in seconds. This value is the simulation sampling interval used to simulate the data.

  System Identification Toolbox uses the sampling interval during model estimation and to set the horizontal axis on time plots. If you transform a time-domain signal to a frequency-domain signal, this sampling interval is used to set the correct frequency scales for the Fourier transforms. For more information about performing Fourier transforms in this toolbox, see the `fft` reference page.

The Import Data dialog box now resembles the following figure.



Import Data

Data Format for Signals

IDDATA or IDFRD/FRD

Workspace Variable

Object: z

Object class: IDDATA (Time Domain)

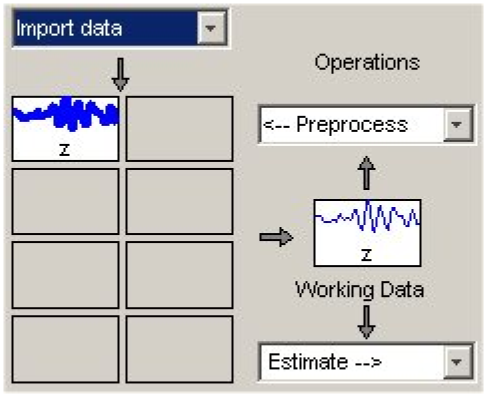
Data Information

Data name: z

Starting time: 0

Sampling interval: 1

3 Click **Import** to add the icon named z to the System Identification Tool window.



4 Click **Close** to close the Import Data dialog box.

Plotting and Preprocessing Data

In this portion of the tutorial, you examine the data and prepare it for system identification. You learn how to:

- Plot the data.
- Subtract the mean values of the input and the output to remove offsets.
- Split the data into two parts. You use one part of the data for model estimation, and the other part of the data for model validation.

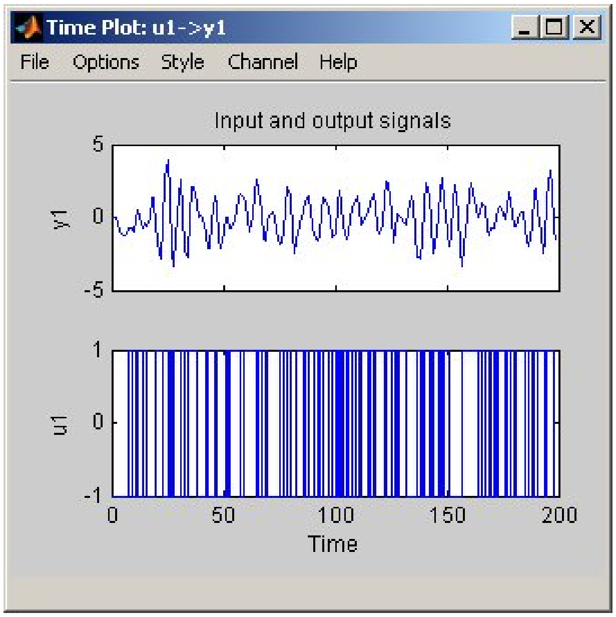
The reason you subtract the mean values from each signal is because, typically, you build linear models that describe the responses for deviations from a physical equilibrium. With steady-state data, it is reasonable to assume that the starting levels of the signals correspond to such an equilibrium. Thus, you can seek models around zero without modeling the absolute equilibrium levels in physical units.

You must have already imported data into the System Identification Tool, as described in “Importing Data Objects into the System Identification Tool” on page 4-7.

Tip For information about other types of preprocessing, such as resampling and filtering the data, see the topics on plotting and preprocessing data in the *System Identification Toolbox User's Guide*.

---

1. In the System Identification Tool window, select the **Time plot** check box to open the Time Plot window.

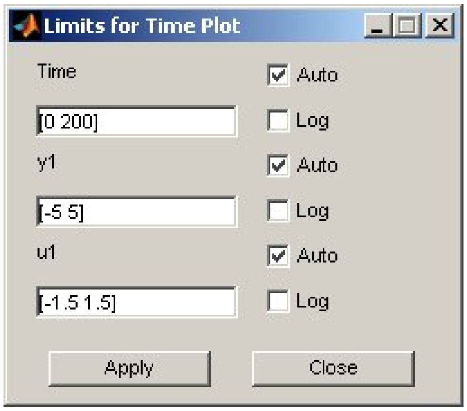
   

   The bottom axes show the simulated input data—a random binary sequence, and the top axes show the simulated output data.

   The next two steps demonstrate how to modify the axis limits in the plot.

2. To modify the vertical-axis limits for the input data, select **Options > Set axes limits**.

3 In the Limits for Time Plot dialog box, set the new vertical axis limit of the input data channel **u1** to [-1.5 1.5]. Click **Apply** and **Close**.



Limits for Time Plot

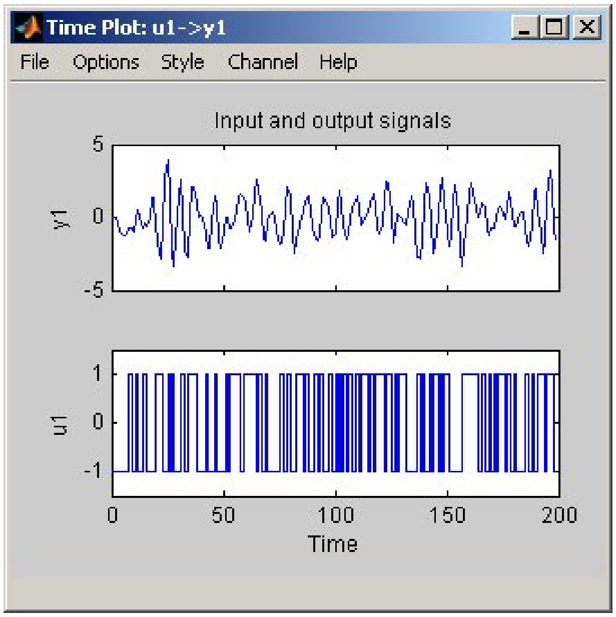
Time: [0 200] — [✓ Auto] [ ] Log

y1: [-5 5] — [✓ Auto] [ ] Log

u1: [-1.5 1.5] — [✓ Auto] [ ] Log

Note The other two fields, **Time** and **y1**, let you set the axis limits for the time axis and the output channel axis, respectively. In the Limits for Time Plot dialog box, you can also modify each axis to be logarithmic or linear.

The following figure shows the time plot.



4 In the System Identification Tool window, select **<--Preprocess > Quick start** to simultaneously perform the following four actions:

- Subtract the mean value from each channel.
- Split the data into two parts.
- Specify the first part of the data as estimation data (or **Working Data**).
- Specify the second part of the data as **Validation Data**.

Estimating Second-Order Process Models with Complex Poles

In this section...

Estimating an Initial Model

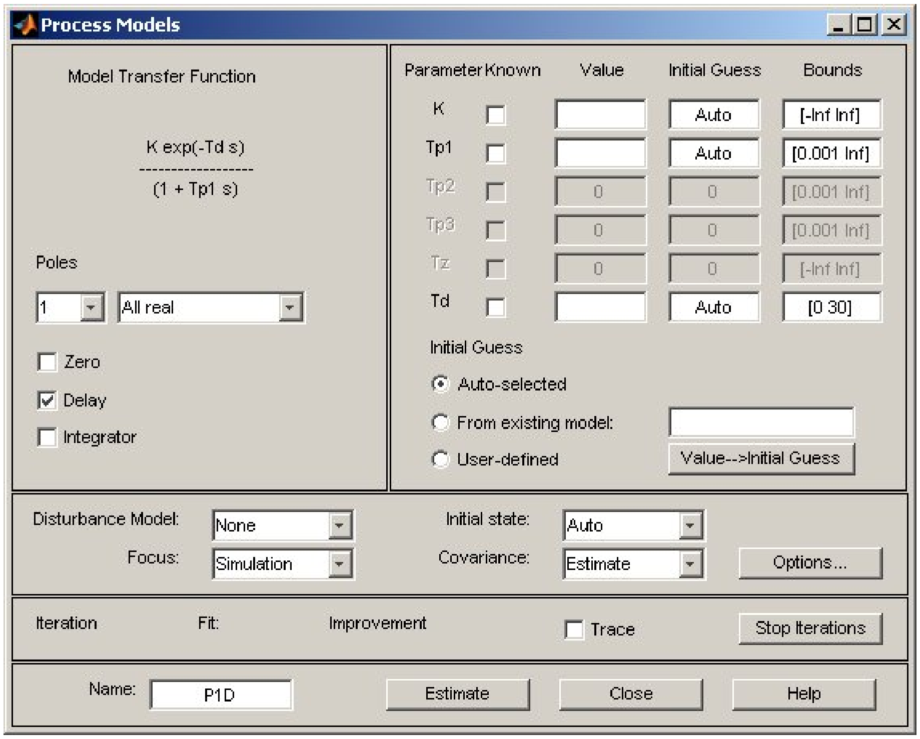
In this portion of the tutorial, you estimate process models with this structure:

$$G(s) = \frac{K}{\left(1 + 2\xi T_w s + {T_w}^2 s^2\right)} e^{-T_d s}$$

This model provides a good starting point for guiding the next step in the identification process.

You must have already prepared the data for estimation, as described in "Plotting and Preprocessing Data" on page 4-10.

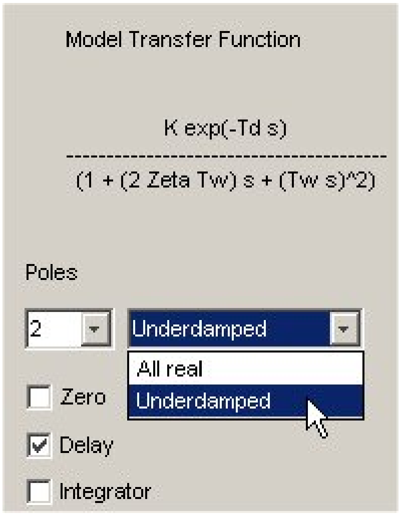
1 In the System Identification Tool window, select **Estimate > Process models** to open the Process Models dialog box.



2 In the **Model Transfer Function** area, specify the following options:

- Under **Poles**, select `2` and `Underdamped`.

  This selection updates the Model Transfer Function to a second-order model structure that can contain complex poles.



Model Transfer Function

```
        K exp(-Td s)
-------------------------------------
 (1 + (2 Zeta Tw) s + (Tw s)^2)
```

Poles: 2, Underdamped

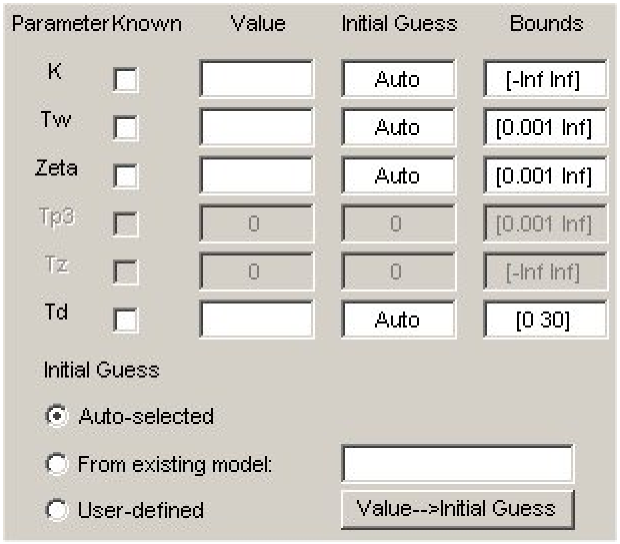
Zero [ ] Delay [✓] Integrator [ ]

The **Parameter** area now shows four active parameters: `K`, `Tw`, `Zeta`, and `Td`. By default, the model **Name** is set to the acronym `P2DU`, which indicates two poles (`P2`), the presence of a delay (`D`), and underdamped modes (`U`).

Note You can edit the model name. Choose a model name that is unique in the System Identification Tool window.

- Make sure that the **Zero** and **Integrator** check boxes are cleared to exclude a zero and an integrator (self-regulating process) from the model.

3 In the **Initial Guess** area, select `Auto-selected` to calculate the initial parameter values during the estimation. The **Initial Guess** column in the Parameter table displays `Auto`.



Parameter	Known	Value	Initial Guess	Bounds
K	☐		Auto	[-Inf Inf]
Tw	☐		Auto	[0.001 Inf]
Zeta	☐		Auto	[0.001 Inf]
Tp3	☐	0	0	[0.001 Inf]
Tz	☐	0	0	[-Inf Inf]
Td	☐		Auto	[0 30]

Initial Guess

[✓ Auto-selected]

From existing model:

User-defined

4 Keep the default **Bounds** values, which specify the minimum and maximum values of each parameter.

When you know the range of possible values for a parameter, type these values into the corresponding **Bounds** field to help the estimation algorithm.

5 Keep the defaults for the estimation algorithm settings:

- **Disturbance Model** — `None` means that the algorithm does not estimate the noise model. This option also sets the **Focus** to `Simulation`.
- **Focus** — `Simulation` means that the estimation algorithm does not use the noise model to weigh the relative importance of how closely to fit the data in various frequency ranges. Instead, the algorithm uses the input spectrum in a particular frequency range to weigh the relative importance of the fit in that frequency range.

Tip The `Simulation` setting is optimized for data that has a high signal-to-noise ratio and for when you plan to use your model for simulation applications. If your system contains significant noise and you want to either model the noise or improve parameter estimates using the noise model, then select `Prediction`.

- **Initial state** — `Auto` means that the algorithm analyzes the data and chooses the optimum method for handling the initial state of the system. If you get poor results, you might try setting a specific method for handling initial states, rather than choosing it automatically.
- **Covariance** — `Estimate` means that the algorithm computes parameter uncertainties that display on plots as model confidence regions.

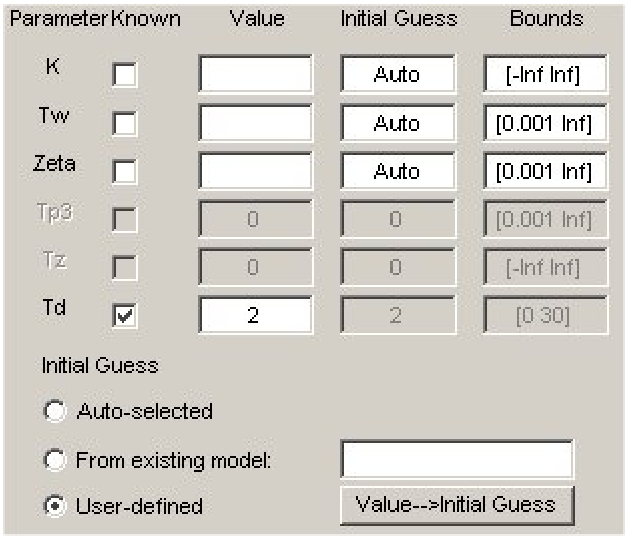
6 Click **Estimate**. This selection adds the model `P2DU` to the System Identification Tool window.

Tips for Specifying Known Parameters

If you know a parameter value exactly, type this value in the **Initial Guess** column.

When you know the value of a parameter approximately, you can help the estimation algorithms by entering an initial value in the **Initial Guess** column. In this case, keep the **Known** check box cleared to allow the estimation to fine-tune this initial guess.

For example, to fix the time-delay value `Td` at 2s, you can type the value into **Value** field of the Parameter table in the Process Models dialog box and select the corresponding **Known** check box.



Parameter	Known	Value	Initial Guess	Bounds
K	☐		Auto	[-Inf Inf]
Tw	☐		Auto	[0.001 Inf]
Zeta	☐		Auto	[0.001 Inf]
Tp3	☐	0	0	[0.001 Inf]
Tz	☐	0	0	[-Inf Inf]
Td	✓	2	2	[0 30]

Initial Guess

- ○ Auto-selected
- ○ From existing model:
- ● User-defined

Validating the Initial Model

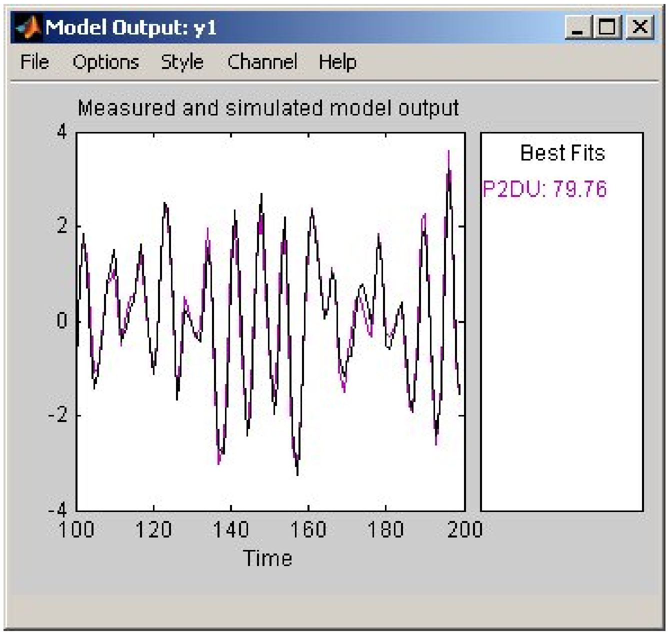
In this portion of the tutorial, you generate the following two plots to examine the initial model you created in “Estimating an Initial Model” on page 4-14:

- Comparison of the model output and the measured output on a time plot
- Autocorrelation of the output residuals, and cross-correlation of the input and the output residuals

Examining Model Output

A good model is the simplest model that best explains the dynamics and successfully simulates or predicts the output for different inputs. Use the model-output plot to check how well the models output matches the measured output in the validation data set.

To generate the model-output plot, select the **Model output** check box in the System Identification Tool window.



System Identification Toolbox uses input validation data as input to the model, and plots the simulated output on top of the output validation data. The preceding plot shows that the model output agrees with the validation-data output.

The **Best Fits** area of the Model Output plot shows the agreement (in percent) between the model output and the validation-data output.

Recall that the sample data is simulated using the following second-order system with underdamped modes (complex poles), and has a peak response at 1 rad/s:

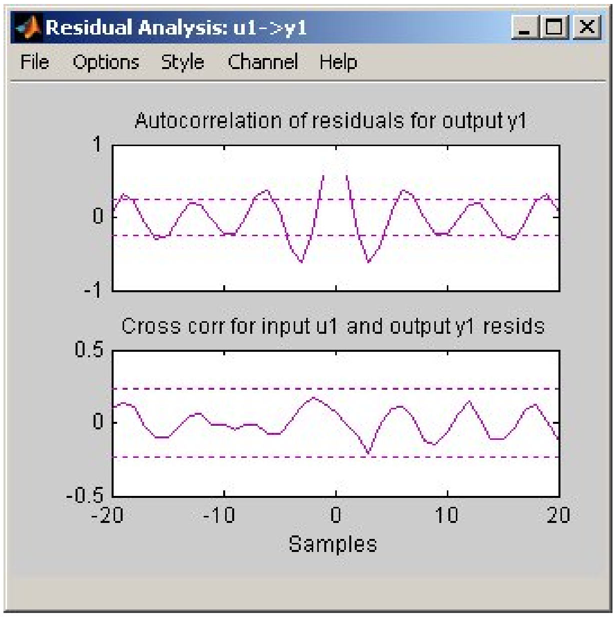
$$G(s) = \frac{1}{1 + 0.2s + s^2} e^{-2s}$$

Because the data includes noise at the input during the simulation, the estimated model cannot exactly reproduce the model used to simulate the data.

Examining Model Residuals

In addition to comparing model output to measured output, you can validate a model by checking the behavior of its residuals.

To generate a Residual Analysis plot, select the **Model resids** check box in the System Identification Tool window.


Residual Analysis: u1->y1
File
Options
Style
Channel
Help
Autocorrelation of residuals for output y1
1
0
-1
Cross corr for input u1 and output y1 resids
0.5
0
-0.5
-20
-10
0
10
20
Samples

The top axes show the autocorrelation of residuals for the output (whiteness test). The horizontal scale is the number of lags, or the difference between the time steps that are correlated. Any fluctuations within the confidence interval are considered to be insignificant. A good model should have a residual autocorrelation function within the confidence interval, indicating that the residuals are uncorrelated. However, in this example, the residuals appear to be correlated.

The bottom axes show the cross-correlation of the residuals with the input. A good model should have residuals uncorrelated with past inputs (independence test). Evidence of correlation indicates that the model does not describe how a portion of the output relates to the corresponding input. For example, when there is a peak outside the confidence interval for lag k, this means that the contribution to the output $y(t)$ that originates from the input $u(t-k)$ is not properly described by the model. In this example, there is no correlation between the residuals and the inputs.

Thus, residual analysis indicates that this process model is good, but that there might be a need for a noise model.

Refining the Process Model

In this section...

Estimating Models with Modified Settings

In this portion of the tutorial, you modify the estimation algorithm and include a noise model to improve the model results.

Note The Process Models dialog box should still be open. If you closed it, repeat the procedure in "Estimating an Initial Model" on page 4-14.

---

1 In the Process Models dialog box, modify the following settings:

   - **Focus** — Set to `Prediction` to specify that the estimation algorithm use the noise model to weigh the relative importance of how closely to fit the data in various frequency ranges. The presence of (high-frequency) noise results in the algorithm assigning less importance to fitting the high-frequency portions of the data.

   - **Disturbance Model** — Set to `Order 1` to estimate a noise model H as a continuous-time, first-order ARMA model:

     $$y = Gu + He$$

     where e is white noise.

   - **Name** — Edit the model name to `P2DUe1` to generate a model with a unique name in the System Identification Tool window.

2 Click **Estimate**.

3 In the Process Models dialog box, set the **Disturbance Model** to `Order 2` to estimate a second-order noise model.

4 Edit the **Name** field to `P2DUe2` to generate a model with a unique name in the System Identification Tool window.

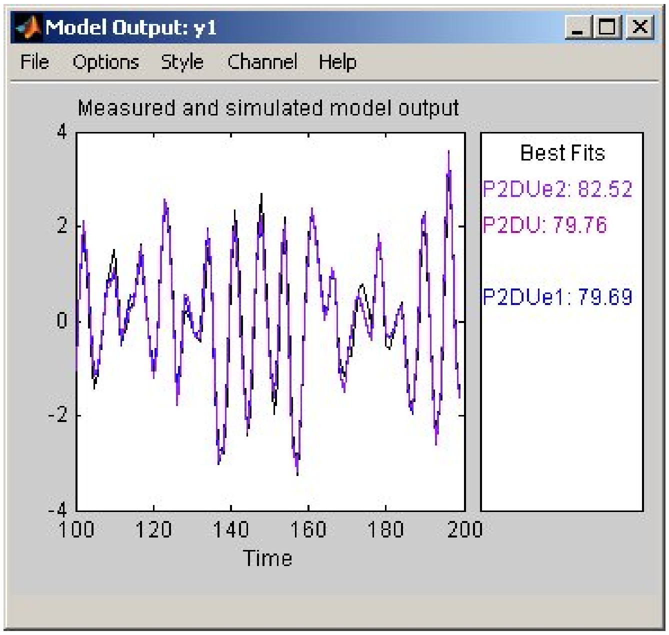
5 Click **Estimate**.

Comparing Models

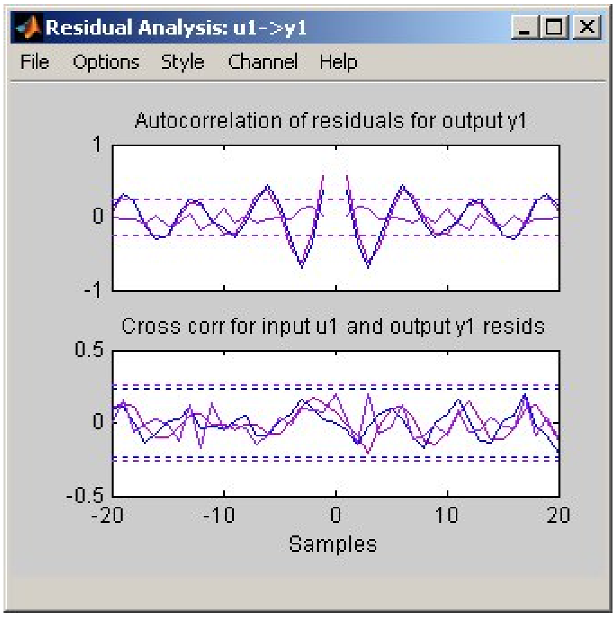
The Model Output and the Residual Analysis windows dynamically update to include the two new models. In this portion of the tutorial, you use these plots to compare the estimated models.

Note If you closed these plots, you can reopen them by selecting the **Model output** and the **Model resids** check boxes in the System Identification Tool window.

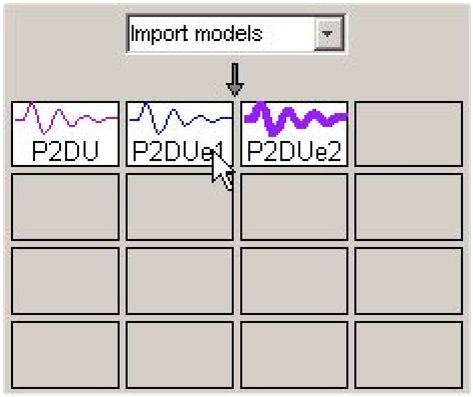
The following Model Output plot shows that the `P2DUe2` model has a better performance than the other two models. However, all three models agree with the validation-data output.



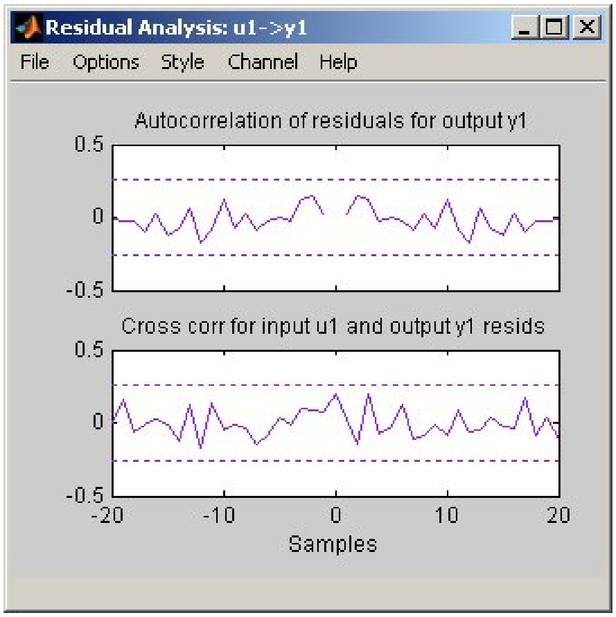
Futhermore, P2DUe2 falls well within the confidence bounds on the Residual Analysis plot.


Residual Analysis: u1->y1
File Options Style Channel Help
Autocorrelation of residuals for output y1
1
0
-1
Cross corr for input u1 and output y1 resids
0.5
0
-0.5
-20
-10
0
10
20
Samples

To view residuals for P2DUe2 only, remove models P2DU and P2DUe1 from the Residual Analysis plot by clicking the corresponding icons in the System Identification Tool window.



The Residual Analysis plot updates, as shown in the following figure.



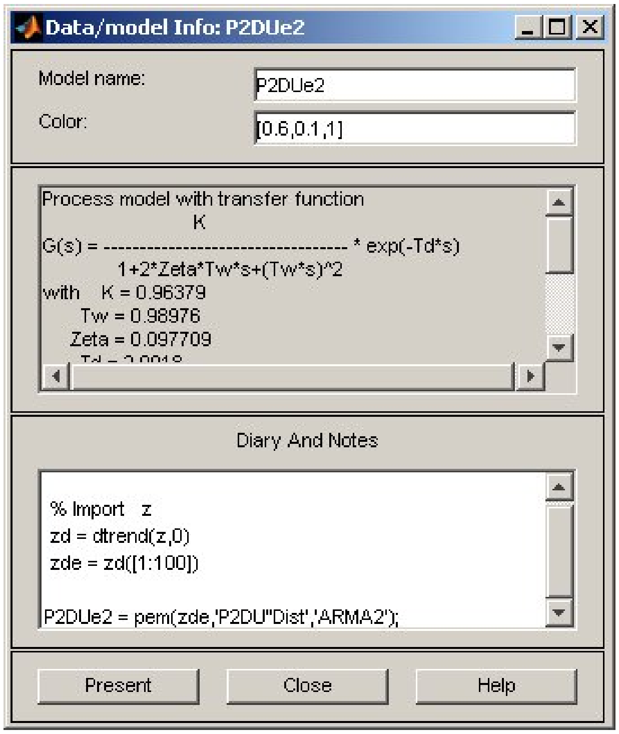
The whiteness test for `P2DUe2` shows that the residuals are uncorrelated, and the independence test shows no correlation between the residuals and the inputs. These tests indicate that `P2DUe2` is a good model.

Viewing Process Model Parameters

In this section...

Viewing Model Parameter Values

You can view the numerical parameter values and other information about the model `P2DUe2` by right-clicking the model icon in the System Identification Tool window. The Data/model Info dialog box opens.



Model name: P2DUe2

Color: [0.6,0.1,1]

```
Process model with transfer function
                    K
G(s) = --------------------------------- * exp(-Td*s)
         1+2*Zeta*Tw*s+(Tw*s)^2
with    K = 0.96379
       Tw = 0.98976
     Zeta = 0.097709
```

```
% Import   z
zd = dtrend(z,0)
zde = zd([1:100])

P2DUe2 = pem(zde,'P2DU''Dist','ARMA2');
```

The noneditable area of the dialog box lists the model coefficients that correspond to the following model structure:

$$G(s) = \frac{K}{\left(1 + 2\xi T_w s + T_w{}^2 s^2\right)} e^{-T_d s}$$

For the model `P2DUe2`:

- `K` is `0.96379`.
- `Tw` is `0.98976`.
- `Zeta` is `0.097709`.
- `Td` is `2.0018`.

These coefficients are agree with the model used to simulate the data:

$$G(s) = \frac{1}{1 + 0.2s + s^2} e^{-2s}$$

`P2DUe2` also includes an additive noise term, where H is a second-order ARMA model and e is white noise:

$$y = Gu + He$$

The Data/model Info dialog box gives the noise model H as a ratio of two polynomials, `C(s)/D(s)`, where:

```
C(s) = s^2 + 2.03(+-0.06772)s + 2.621(+-0.3984)
D(s) = s^2 + 0.2123(+-0.07437)s + 1.113(+-0.07804)
```

The 1-standard-deviation uncertainty for each set of model parameters is in parentheses next to each parameter value.

Viewing Parameter Uncertainties

To view parameter uncertainties for the system transfer function, click **Present** in the Data/model Info dialog box, and view the information in the MATLAB Command Window.

```
K = 0.96379+-0.018245
Tw = 0.98976+-0.0055579
```

```
   Zeta = 0.097709+-0.0064056
   Td = 2.0018+-0.0025342
```

The 1-standard-deviation uncertainty for each set of model parameters is in parentheses next to each parameter value.

Exporting the Model to the MATLAB Workspace Browser

You can perform further analysis on your estimated models from the MATLAB Workspace browser. For example, if the model is a plant that requires a controller, you can import the model from the MATLAB Workspace browser into Control System Toolbox. To simulate your model in Simulink, perhaps as part of a larger dynamic system, you can import this model as a Simulink block.

The models you create in the System Identification Tool GUI are not automatically available in the MATLAB Workspace browser. To make a model available to other toolboxes, Simulink, and System Identification Toolbox commands, you must export your model from the System Identification Tool to the MATLAB Workspace browser.

To export the `P2DUe2` model, drag it to the **To Workspace** rectangle in the System Identification Tool window. The model now appears in the MATLAB Workspace browser.

Note This model is an object that belongs to the `idproc` class. To learn more about this model object, see the corresponding reference page.

Using Simulink with System Identification Toolbox

In this section...

Preparing Input Data in the MATLAB Workspace Browser

You can create a simple Simulink model that uses blocks from the System Identification Toolbox library to bring the data `z` and the model `P2DUe2` into Simulink.

You must have completed the previous steps in this tutorial to make these variables available in the MATLAB Workspace browser.

Note Simulink must be installed to build the Simulink model.

Because you only need the input channel of `z` for providing input to the model, type the following in the MATLAB Command Window:

```
z_input = z    % Creates a new iddata object.
z_input.y = [] % Sets the output channel
               % to empty.
```

Building the Simulink Model

The following steps guide you through the process of adding blocks to a Simulink model. For more information about working with Simulink models, see the Simulink documentation.

1 In the MATLAB Command Window, type `simulink` at the MATLAB prompt.

2 Select **File > New > Model** to open a new model window.

3 In the Simulink Library Browser window, select the **System Identification Toolbox** library. The right side of the window displays blocks specific to System Identification Toolbox.

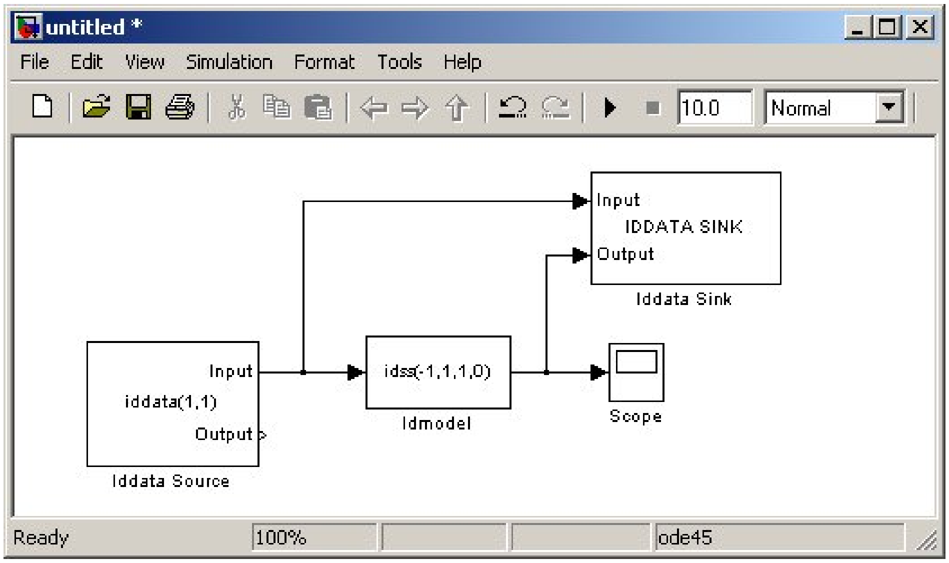
Tip An alternative way to access the System Identification block library is to type `slident` in the MATLAB Command Window.

4 Drag the following System Identification Toolbox blocks to the new model window:

- Iddata Source block
- Idmodel block
- Iddata Sink block

5 In the Simulink Library Browser window, select the **Simulink > Sinks** library, and drag the Scope block to the new model window.

6 In the Simulink model window, connect the blocks until your model resembles the following figure.



In the next section, you configure these blocks to get information from the MATLAB Workspace browser and set the simulation time interval and duration.

Configuring Blocks and Simulation Parameters

The following procedure guides you through the following tasks to configure the model blocks:

- Get data from the MATLAB Workspace browser.
- Set the simulation parameters.

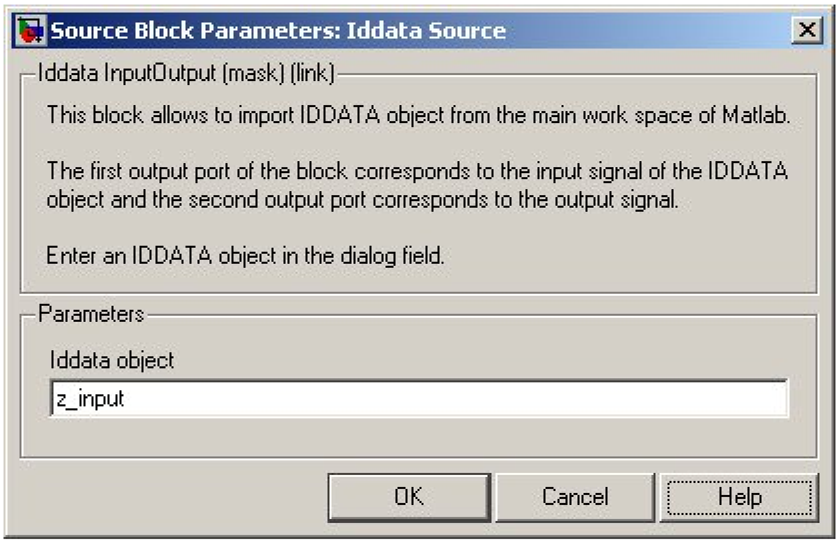
1 In the new model window, select **Simulation > Configuration Parameters**.

2 In the Configuration Parameters dialog box, type 200 in the **Stop time** field. Click **OK**. This value sets the duration of the simulation to 200 seconds.

3 Double-click the Iddata Source block to open the Source Block Parameters: Iddata Source dialog box. Next, type the following variable name in the **Iddata object** field:

```
z_input
```

This variable represents the data object in the MATLAB Workspace browser that contains the input data.



Source Block Parameters: Iddata Source

Iddata InputOutput (mask) (link)

This block allows to import IDDATA object from the main work space of Matlab.

The first output port of the block corresponds to the input signal of the IDDATA object and the second output port corresponds to the output signal.

Enter an IDDATA object in the dialog field.

Parameters

Iddata object

z_input

Tip As a shortcut, you can drag and drop the variable name from the MATLAB Workspace browser to the **Iddata object** field.

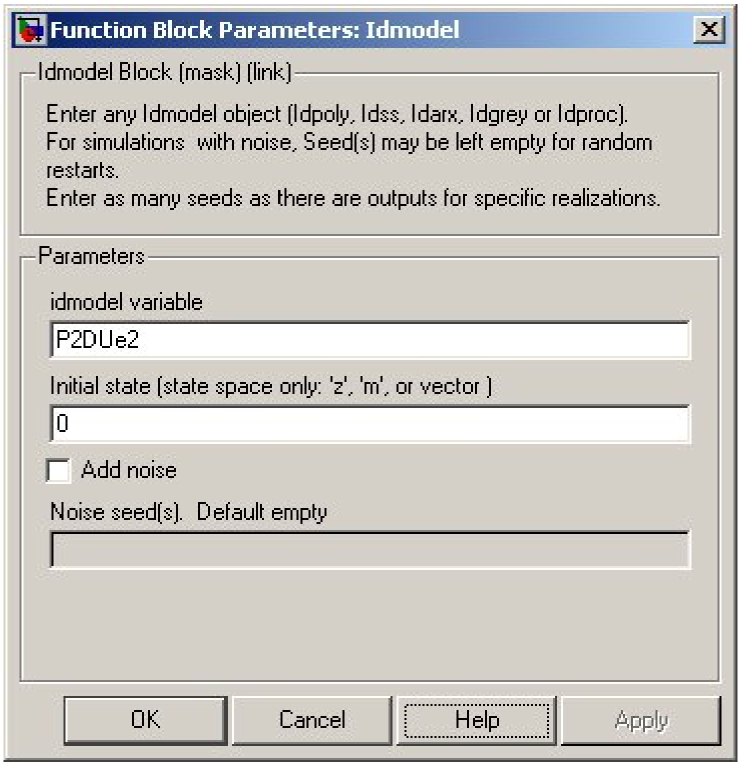
---

4 Click **OK**.

5 Double-click the Idmodel block to open the Function Block Parameters: Idmodel dialog box. Then, type the following variable name in the **idmodel variable** field:

```
P2DUe2
```

This variable represents the name of the process model in the MATLAB Workspace browser.


Function Block Parameters: Idmodel

Idmodel Block (mask) (link)

Enter any Idmodel object (Idpoly, Idss, Idarx, Idgrey or Idproc).
For simulations with noise, Seed(s) may be left empty for random restarts.
Enter as many seeds as there are outputs for specific realizations.

Parameters

idmodel variable: P2DUe2

Initial state (state space only: 'z', 'm', or vector): 0

☐ Add noise

Noise seed(s). Default empty:

6. Clear the **Add noise** check box to exclude noise from the system. Click **OK**.

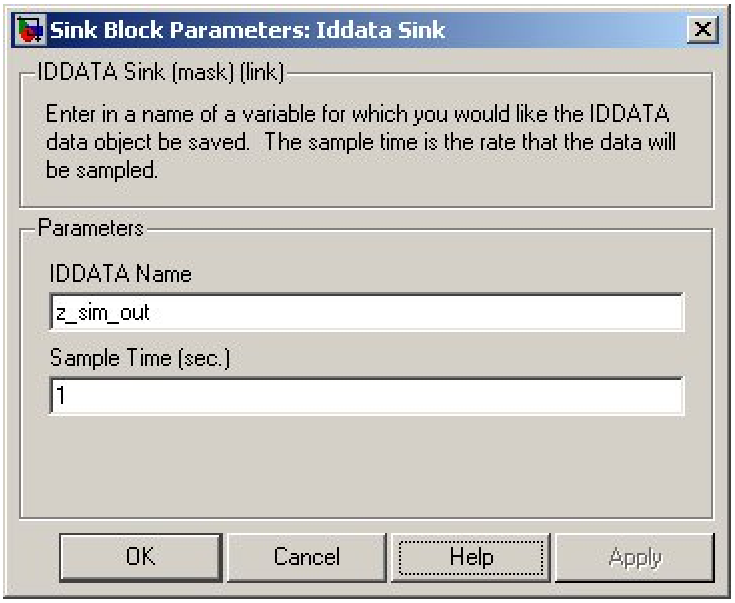
   When **Add noise** is selected, Simulink derives the noise amplitude from the `NoiseVariance` property of the model and adds noise to the model accordingly. The simulation propagates this noise according to the noise model *H* that you estimated with the system dynamics in System Identification Toolbox:

   $$y = Gu + He$$

7. Double-click the Iddata Sink block to open the Sink Block Parameters: Iddata Sink dialog box. Then, type the following variable name in the **IDDATA Name** field:

   ```
   z_sim_out
   ```

8. Type `1` in the **Sample Time (sec.)** field to set the sampling time of the output data to be the same as the sampling time of the input data. Click **OK**.



Sink Block Parameters: Iddata Sink

IDDATA Sink (mask) (link)

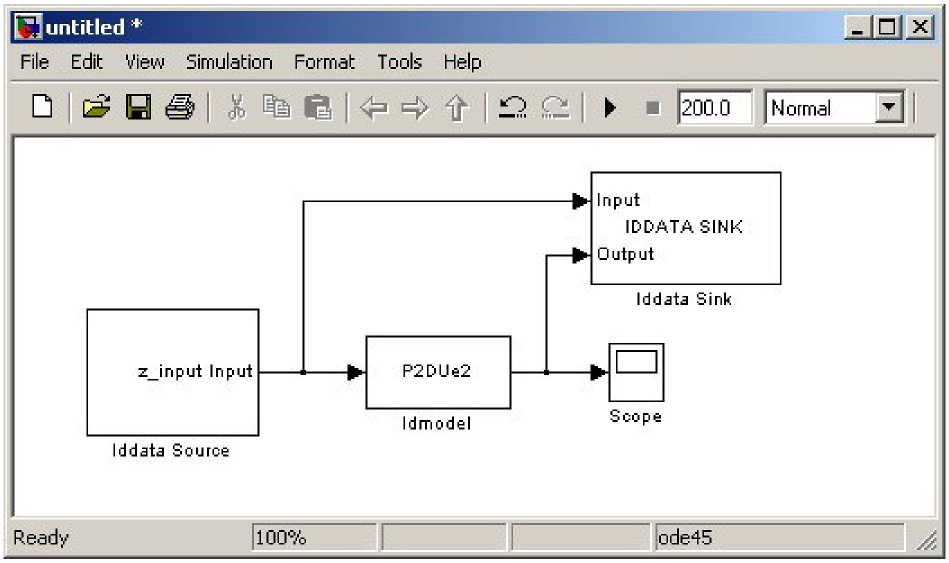
Enter in a name of a variable for which you would like the IDDATA data object be saved. The sample time is the rate that the data will be sampled.

Parameters

IDDATA Name: z_sim_out

Sample Time (sec.): 1

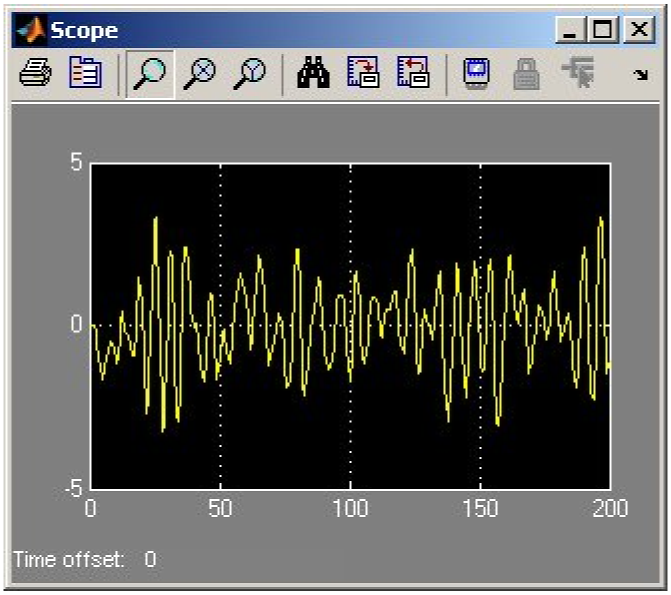
The resulting change to the Simulink model is shown in the following figure.



Running the Simulation

1 In the Simulink model window, select **Simulation > Start**.

2 Double-click the Scope block to display the time plot of the model output.



3 In the MATLAB Workspace browser, notice the variable `z_sim_out` that stores the model output as an `iddata` object. You specified this variable name when you configured the Iddata Sink block.

This variable stores the simulated output of the model and it is now available for further processing and exploration.

Tutorial: Estimating Linear Models Using the Command Line

About This Tutorial

In this section...

Objectives

Estimate and validate linear models from multiple-input/single-output (MISO) data to find the one that best represents your system dynamics.

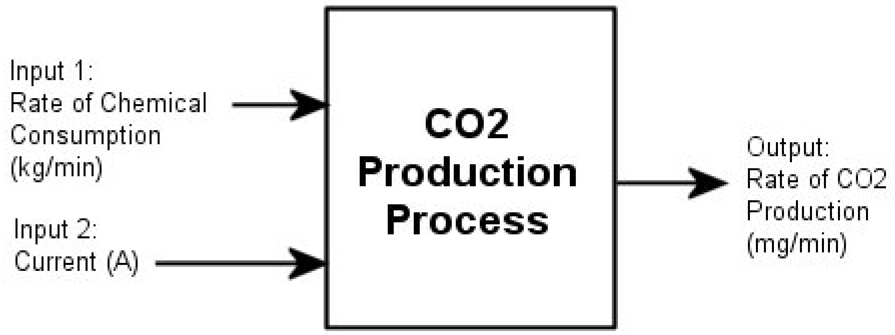
After completing this tutorial, you will be able to accomplish the following tasks using the command line:

- Create data objects to represent data.
- Plot the data.
- Preprocess data by removing offsets from the input and output signals.
- Estimate and validate linear models from the data.
- Simulate and predict model output.

Sample Data

The sample data is the MAT-file `co2data.mat`, which contains two experiments of two-input and single-output (MISO) time-domain data from a steady-state process that the operator perturbed from equilibrium values.

In the first experiment, the operator introduced a pulse wave to both inputs. In the second experiment, the operator introduced a pulse wave to the first input and a step signal to the second input.



Note This tutorial uses time-domain data to demonstrate how you can estimate linear models. This workflow also applies to frequency-domain data.

Preparing Data

In this section...

Loading Data into the MATLAB Workspace Browser

Load the sample data in `co2data.mat` by typing the following command at the MATLAB prompt:

```
load co2data;
```

This command loads the following five variables into the MATLAB Workspace browser:

- `Input_exp1` and `Output_exp1` are the input and output data from the first experiment, respectively.
- `Input_exp2` and `Output_exp2` are the input and output data from the second experiment, respectively.
- `Time` is the time vector from 0 to 1000 minutes, increasing in equal increments of `0.5` min.

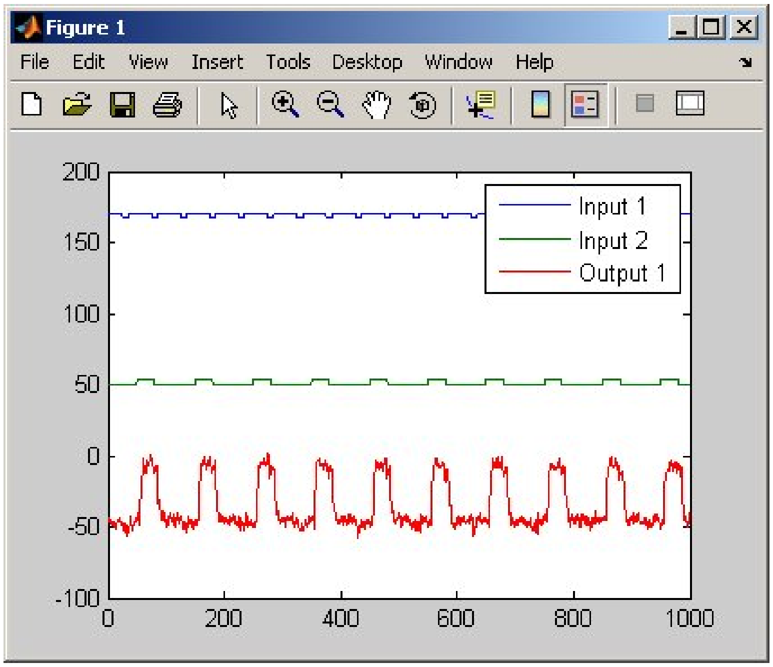
For both experiments, the input data consists of two columns of values. The first column of values is the rate of chemical consumption (in kilograms per minute), and the second column of values is the current (in amperes). The output data is a single column of the rate of carbon-dioxide production (in milligrams per minute).

Plotting the Input/Output Data

You can plot the input and output data from both experiments using the following commands:

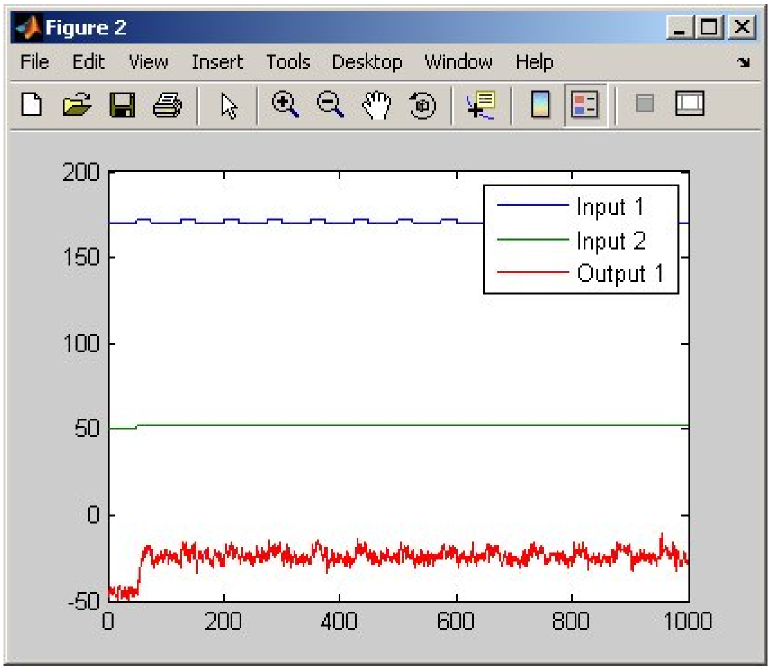
```
% Plot the input and output data from both experiments
plot(Time,Input_exp1,Time,Output_exp1)
legend('Input 1','Input 2','Output 1')
figure
plot(Time,Input_exp2,Time,Output_exp2)
legend('Input 1','Input 2','Output 1')
```

The following plot shows the first experiment, where the operator applies a pulse wave to each input to perturb it from its steady-state equilibrium.



Input and Output Data from Experiment 1

The following plot shows the second experiment, where the operator applies a pulse wave to the first input and a step signal to the second input.



Input and Output Data from Experiment 2

Removing Equilibrium Values from the Data

Plotting the data, as described in "Plotting the Input/Output Data" on page 5-6, shows that the inputs and the outputs have nonzero equilibrium values. In this portion of the tutorial, you subtract equilibrium values from the data.

The reason you subtract the mean values from each signal is because, typically, you build linear models that describe the responses for deviations from a physical equilibrium. With steady-state data, it is reasonable to assume that the starting levels of the signals correspond to such an equilibrium. Thus, you can seek models around zero without modeling the absolute equilibrium levels in physical units.

Zoom in on the plots to see that the earliest moment when the operator applies a disturbance to the inputs occurs after 25 minutes of steady-state conditions (or after the first 50 samples). Thus, the average value of the first 50 samples represents the equilibrium conditions.

Use the following commands to remove the equilibrium values from the inputs and the outputs in both experiments:

```
% Remove the equilibrium values from the inputs and the outputs in both
% experiments:
Input_exp1 = Input_exp1-...
   ones(size(Input_exp1,1),1)*mean(Input_exp1(1:50,:));
Output_exp1 = Output_exp1-...
   mean(Output_exp1(1:50,:));
Input_exp2 = Input_exp2-...
   ones(size(Input_exp2,1),1)*mean(Input_exp2(1:50,:));
Output_exp2 = Output_exp2-...
   mean(Output_exp2(1:50,:));
```

Note The `ones` command replicates the two mean values, one for each input, in a two-dimensional array.

Using Objects to Represent Data for System Identification

The System Identification Toolbox data objects, `iddata` and `idfrd`, encapsulate both data values and data properties into a single entity. System Identification Toolbox commands let you conveniently manipulate these data objects as single entities.

In this portion of the tutorial, you create two `iddata` objects, one for each of the two experiments. You use the data from Experiment 1 for model estimation, and the data from Experiment 2 for model validation. You work with two independent data sets because you use one data set for model estimation and the other for model validation.

Note When two independent data sets are not available, you can split one data set into two parts, assuming that each part contains enough information to adequately represent the system dynamics.

Creating iddata Objects

You must have already loaded the sample data into the MATLAB Workspace browser, as described in "Loading Data into the MATLAB Workspace Browser" on page 6-8.

Use these commands to create two data objects, `ze` and `zv`:

```
% Create two data objects, ze and zv.
Ts = 0.5; % Sampling interval is 0.5 min
ze = iddata(Output_exp1,Input_exp1,Ts);
zv = iddata(Output_exp2,Input_exp2,Ts);
```

`ze` contains data from Experiment 1 and `zv` contains data from Experiment 2. `Ts` is the sampling interval.

The `iddata` constructor requires three arguments for time-domain data and has the following syntax:

```
data_obj = iddata(output,input,sampling_interval);
```

To view the properties of an `iddata` object, use the `get` command. For example, type this command to get the properties of the estimation data:

```
get(ze)
```

MATLAB returns the following data properties and values:

```
           Domain: 'Time'
             Name: []
       OutputData: [2001x1 double]
                y: 'Same as OutputData'
       OutputName: {'y1'}
       OutputUnit: {''}
        InputData: [2001x2 double]
                u: 'Same as InputData'
        InputName: {2x1 cell}
        InputUnit: {2x1 cell}
           Period: [2x1 double]
      InterSample: {2x1 cell}
               Ts: 0.5000
           Tstart: []
 SamplingInstants: [2001x0 double]
         TimeUnit: ''
   ExperimentName: 'Exp1'
            Notes: []
         UserData: []
```

To learn more about these properties, see the `iddata` reference page.

To modify data properties, you can use dot notation or the `set` command. For example, to assign channel names and units that label plot axes, type the following syntax at the MATLAB prompt:

```
% Set time units to minutes
ze.TimeUnit = 'min';
% Set names of input channels
ze.InputName = {'ConsumptionRate','Current'};
% Set units for input variables
ze.InputUnit = {'kg/min','A'};
% Set name of output channel
ze.OutputName = 'Production';
% Set unit of output channel
ze.OutputUnit = 'mg/min';

% Set validation data properties
zv.TimeUnit = 'min';
zv.InputName = {'ConsumptionRate','Current'};
zv.InputUnit = {'kg/min','A'};
zv.OutputName = 'Production';
zv.OutputUnit = 'mg/min';
```

You can verify that the `InputName` property of `ze` is changed, or "index into" this property, by typing the following syntax:

```
ze.inputname
```

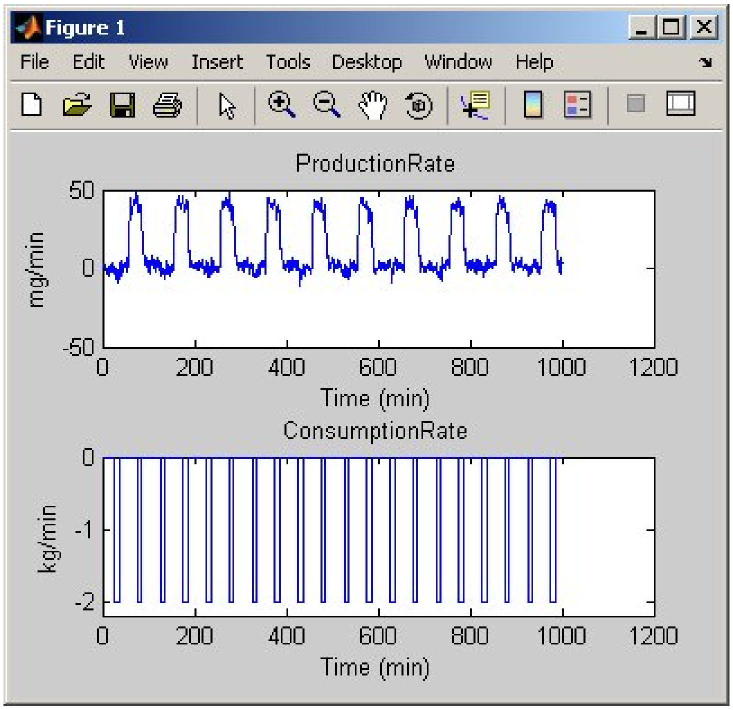
Tip Property names, such as `InputUnit`, are not case sensitive. You can also abbreviate property names that start with `Input` or `Output` by substituting `u` for `Input` and `y` for `Output` in the property name. For example, `OutputUnit` is equivalent to `yunit`.

Plotting the Data

You can plot `iddata` objects using the MATLAB `plot` command:

```
plot(ze)     % Plot the estimation data
```

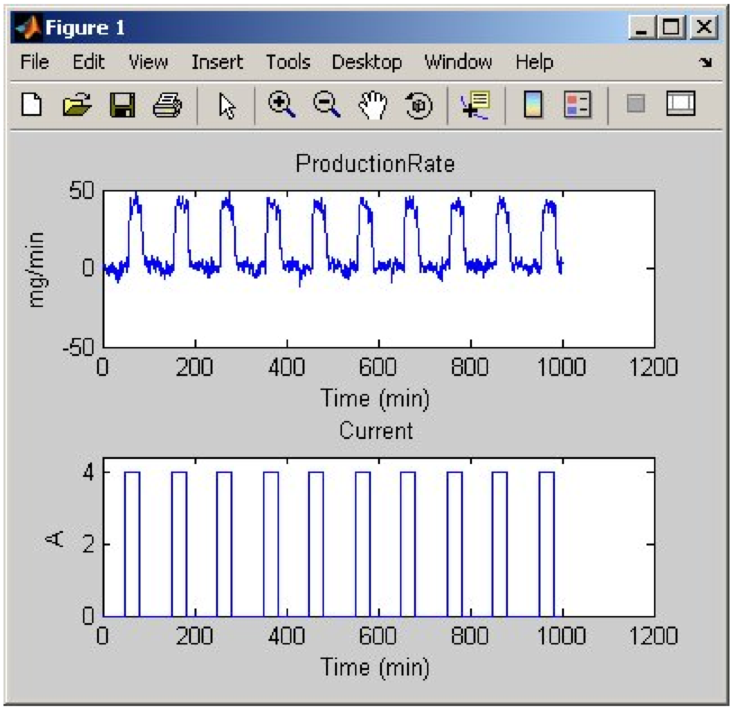
This opens the following plot. The bottom axes show the first input `ConsumptionRate`, and the top axes show the output `ProductionRate`.



Input 1 and Output for ze

For multivariable data, only one input/output pair appears on the plot at a time. To view the second input `Current`, select the MATLAB Figure window, and press **Enter** to update the plot.

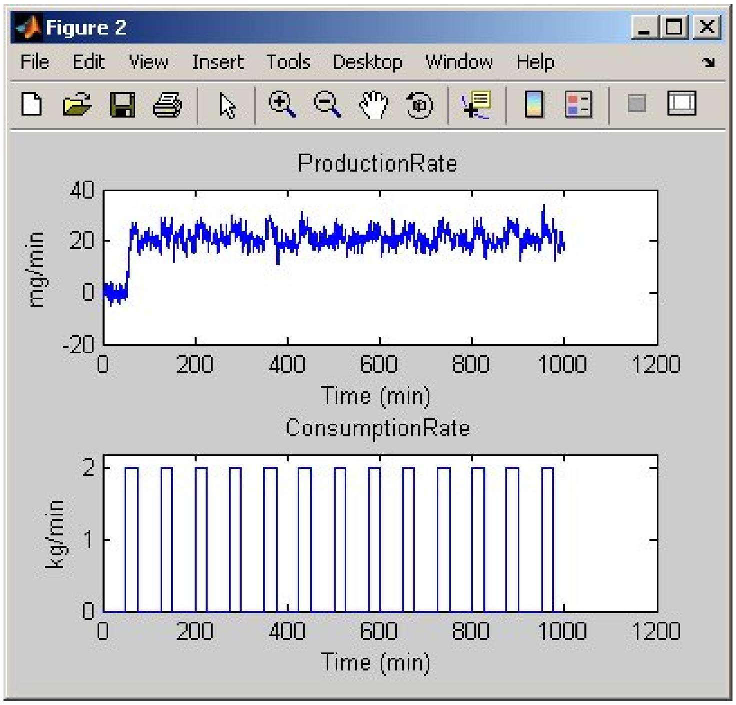
Tip For plots of data with multiple inputs and outputs, press **Enter** to view the next input/output pair.



Input 2 and Output for ze

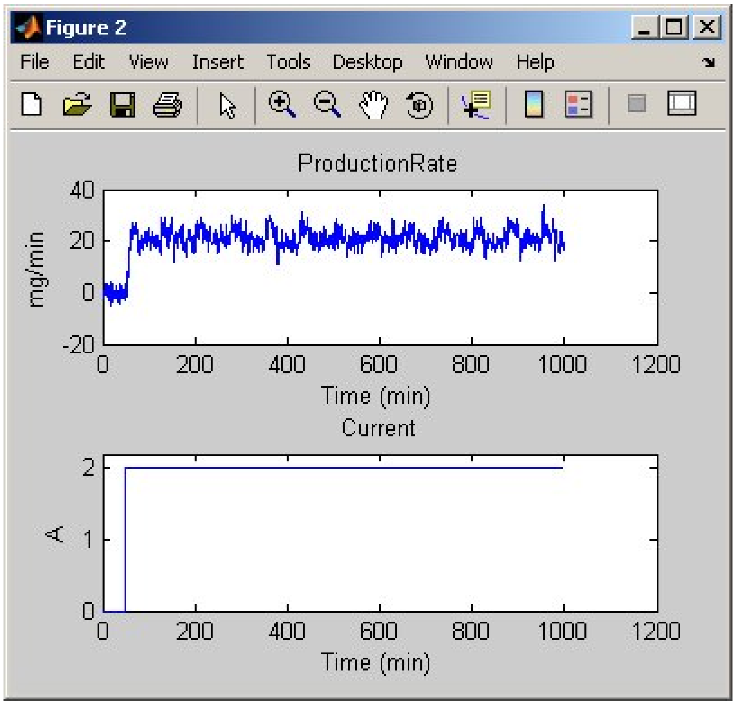
To plot the validation data in a new MATLAB Figure window, type the following commands at the MATLAB prompt:

```
figure   % Open a new MATLAB Figure window
plot(zv) % Plot the validation data
```



Input 1 and Output for zv

Select the MATLAB Figure window, and press **Enter** to view the second input on the plot.



Input 2 and Output for zv

Zoom in on the plots to see that the process amplifies the first input (`ConsumptionRate`) by a factor of 2, and amplifies the second input (`Current`) by a factor of 10.

Selecting a Subset of the Data

Before you begin, create a subset of 1000 samples from the original estimation and validation data sets to speed up the calculations:

```
Ze1 = ze(1:1000);
Zv1 = zv(1:1000);
```

For more information about indexing into `iddata` objects, see the corresponding reference page.

Estimating Nonparametric Models

In this section...

Why Estimate Nonparametric Models?

Nonparametric models are frequency-response and step-response models, which can help you understand the dynamic characteristics of your system. These models are not represented by a compact mathematical formula with adjustable parameters. Instead, they consist of data tables.

In this portion of the tutorial, you estimate nonparametric models using the data set `ze`. You must have already created `ze`, as described in "Creating iddata Objects" on page 5-9.

The response plots from these models provide you with the following insights into your system:

- The response from the first input to the output might be a second-order function.
- The response from the second input to the output might be a first-order or an overdamped function.

Estimating the Frequency Response

System Identification Toolbox provides three functions for estimating the frequency response:

- `etfe` computes the empirical transfer function using Fourier analysis.
- `spa` estimates the transfer function using spectral analysis for a fixed frequency resolution.
- `spafdr` lets you specify a variable frequency resolution for estimating the frequency response.

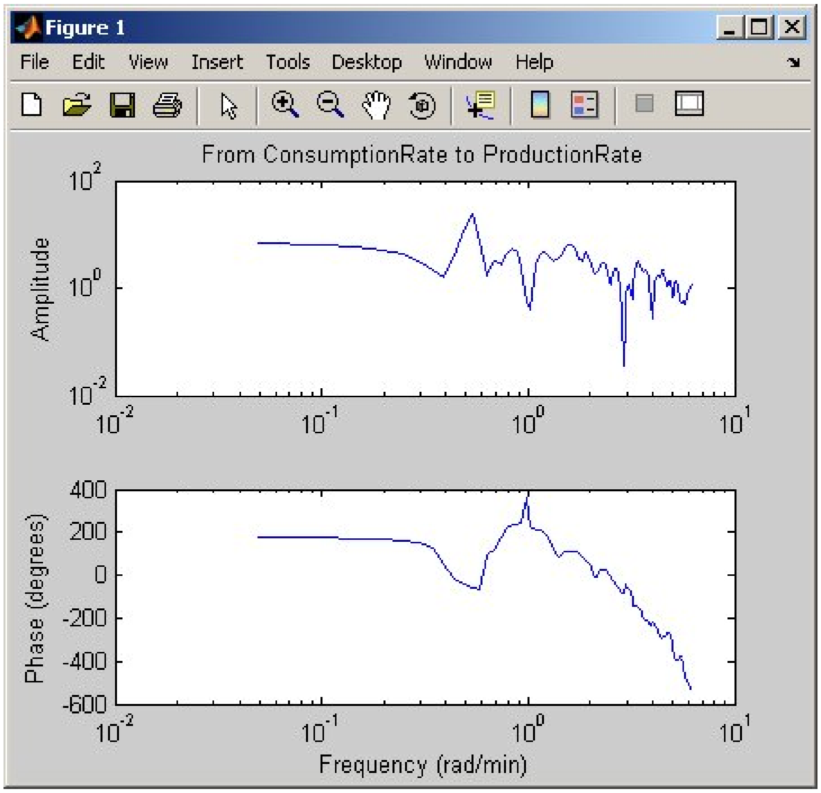
Use the spa command to estimate the frequency response:

```
Ge=spa(ze);
```

To plot the frequency response as a Bode plot, type the following command at the MATLAB prompt:

```
bode(Ge)
```

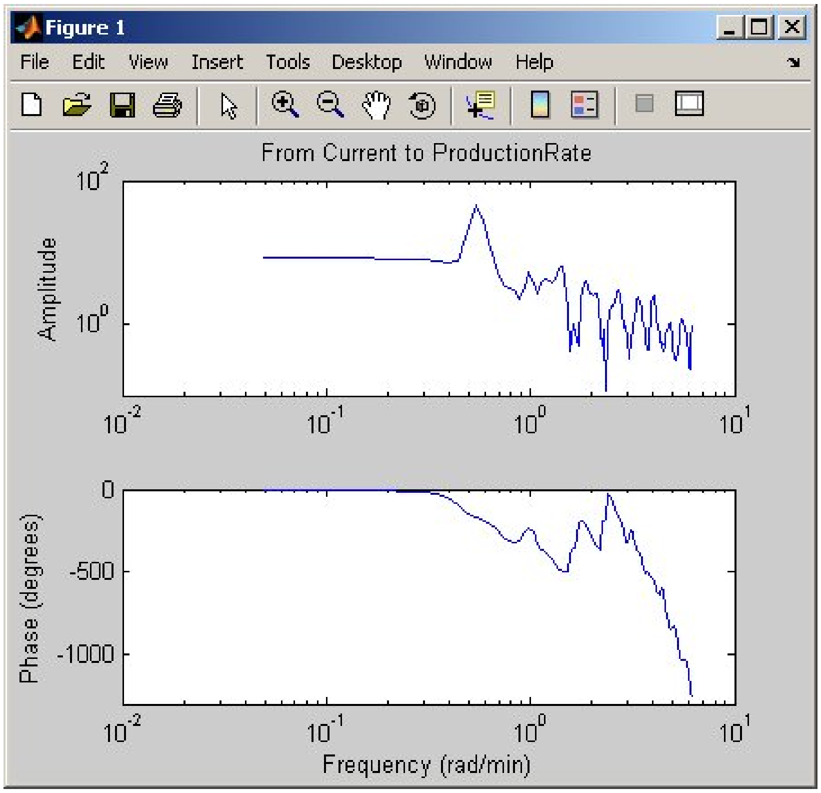
This command produces the following plot.



Frequency Response for the First Input-Output Path

The amplitude peaks at the frequency of about 0.7 rad/s, which suggests a possible resonant behavior (complex poles) for the first input-to-output combination—ConsumptionRate to ProductionRate.

To view the second input `Current`, select the MATLAB Figure window, and press **Enter**. The input/output pair is displayed, as shown in the following figure.



Frequency Response for the Second Input-Output Path

In both plots, the phase rolls off rapidly, which suggests a time delay for both input/output combinations.

Tip When your data contains multiple inputs and outputs, press **Enter** to view the next input/output pair.

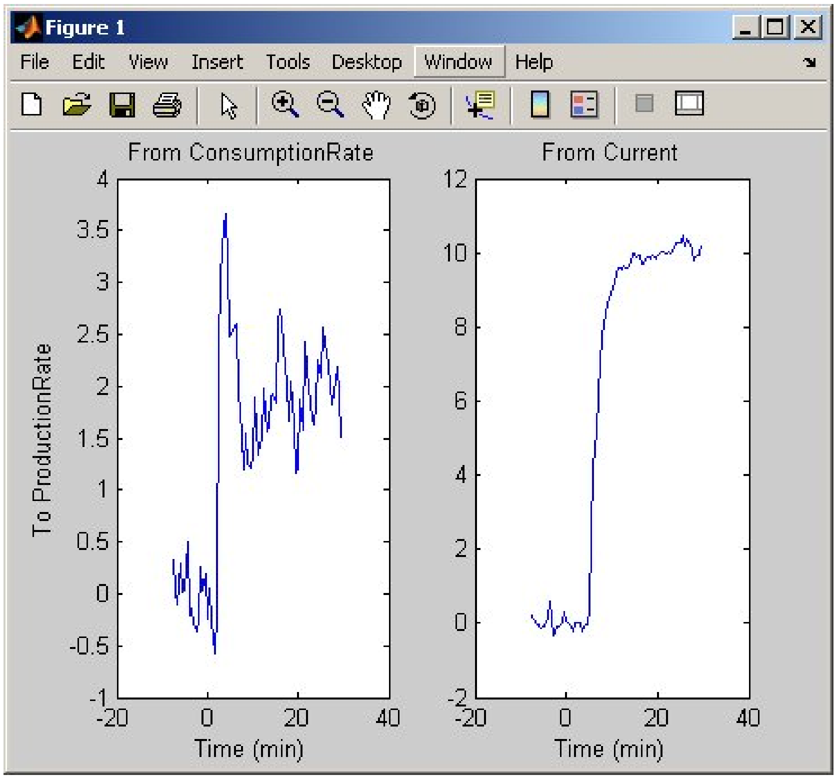
Estimating the Step Response

To estimate the step response from the data, use the `step` command with the following arguments:

```
step(ze,30)
```

The first `step` argument is the name of the data object. The second argument is the duration of the step input in the time units you specified (minutes).

This calculation produces the following plot.



Step Response from Both Inputs to the Output

The step response for the first input/output combination suggests an overshoot, which indicates the presence of an underdamped mode (complex poles) in the physical process.

The step response from the second input to the output shows no overshoot, which indicates either a first-order response or a higher order response with real poles (overdamped response).

The step-response plot indicates a nonzero delay in the system, which is consistent with the rapid phase roll-off you got in the Bode plot you created in “Estimating the Frequency Response” on page 5-17.

Estimating Delays in the System

In this section...

Why Estimate Delays?

To estimate parametric black-box models using System Identification Toolbox, you must specify the delay and the model order as inputs.

If you do not know the input/output delays for your system from the experiment, use System Identification Toolbox to estimate the delay.

Next, as you explore different model structures, you can specify delay values that are slight variations around the initial delay estimate.

Estimating Delays Using an ARX Model

In the case of single-input systems, you can read the delay on the impulse-response plot. However, in the case of multiple-input systems, such as the one in this tutorial, you might be unable to tell which input caused the initial change in the output and you can use the `delayest` command instead.

The `delayest` command estimates the time delay in a dynamic system by estimating a low-order, discrete-time ARX model with a range of delays, and then choosing the delay that corresponding to the best fit. The ARX model structure is one of the simplest black-box parametric structures.

In discrete-time, the ARX structure is a difference equation with the following form:

$$y(t)+a_1y(t-1)+\ldots+a_{na}y(t-n_a)=$$
$$\mathrm{b}_1u(t-n_k)+\ldots+b_{nb}u(t-n_k-n_b+1)+e(t)$$

y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, n_a is the number of poles, n_b is the number of *b* parameters (equal to the number of zeros plus 1), n_k is the delay (the number of samples before the input affects output of the system), and *e(t)* is the white-noise disturbance.

`delayest` assumes that n_a=n_b=`2` and that the noise *e* is white or insignificant, and estimates n_k.

To estimate the delay in this system, type the following command at the MATLAB prompt:

```
delayest(ze)
```

MATLAB responds with the following:

```
ans =

     5    10
```

System Identification Toolbox gives two answers because there are two inputs: the estimated delay for the first input is `5` data samples, and the estimated delay for the second input is `10` data samples. Because the sampling interval for the experiments is `0.5` min, this corresponds to a `2.5`-min delay before the first input affects the output, and a `5.0`-min delay before the second input affects the output.

Alternative Methods for Estimating Delays

There are two alternative methods for estimating the time delay in the system:

- Plot the time plot of the input and output data and read the time difference between the first change in the input and the first change in the output. This method is practical only for single-input/single-output system; in the case of multiple-input systems, you might be unable to tell which input caused the initial change in the output.

- Plot the impulse response of the data with a 1-standard-deviation confidence region. You can estimate the time delay using the time when the impulse response is first outside the confidence region.

Estimating Model Orders Using a Simple ARX Structure

In this section...

Why Estimate Model Order?

Model order is one or more integers that define the complexity of the model. In general, model order is related to the number of poles, the number of zeros, and the response delay (time in terms of the number of samples before the output responds to the input). The specific meaning of model order depends on the model structure.

To compute parametric black-box models using System Identification Toolbox, you must provide the model order as an input. If you do not know the order of your system, you can estimate it.

After completing the steps in this section, you get the following results:

- For the first input/output combination: n_a=2, n_b=2, and the delay n_k=5.
- For the second input/output combination: n_a=1, n_b=1, and the delay n_k=10.

If you do not know the model order, use System Identification Toolbox to estimate the order.

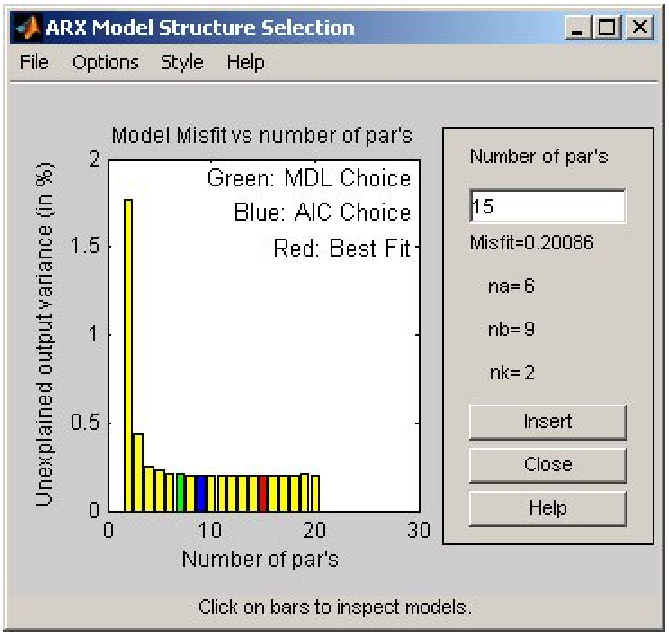
Then, as you explore different model structures, you can specify model-order values that are slight variations around the initial estimate.

Commands for Estimating the Model Order

In this portion of the tutorial, you use `struc`, `arxstruc`, and `selstruc` to estimate a collection of ARX models with different combination of orders, and select the best orders based on the quality of the fit to the data. This approach provides a good initial guess of the model order that captures the system dynamics.

The following list describes the results of using each command:

- `struc` creates a matrix of possible model-order combinations for a specified range of n_a, n_b, and n_k values.
- `arxstruc` takes the output from `struc`, systematically estimates an ARX model for each model order, and compares the model output to the measured output. `arxstruc` returns the *loss function* for each model, which is the normalized sum of squared prediction errors.
- `selstruc` takes the output from `arxstruc` and opens the ARX Model Structure Selection window, which resembles the following figure, to help you choose the model order.



Number of par's

15

Misfit=0.20086

na=6

nb=9

nk=2

Click on bars to inspect models.

You use the preceding plot to select the best-fit model. The horizontal axis is the total number of parameters:

$$\text{Number of parameters} = n_a + n_b$$

For the ARX model, n_a is the number of poles, n_b is the number of *b* parameters (equal to the number of zeros plus 1), and n_k is the delay.

The vertical axis, called **Unexplained output variance (in %)**, is the portion of the output not explained by the model—the ARX model prediction error for a specific number of parameters. The *prediction error* is the sum of the squares of the differences between the validation data output and the model output.

Three rectangles are highlighted on the plot in green, blue, and red. Each color indicates a type of best-fit criterion, as follows:

- Red — Best fit minimizes the sum of the squares of the difference between the validation data output and the model output. This rectangle indicates the overall best fit.
- Green — Best fit minimizes Rissanen MDL criterion.
- Blue — Best fit minimizes Akaike AIC criterion.

In this tutorial, the **Unexplained output variance (in %)** value remains approximately constant for the combined number of parameters from 4 to 20. Such constancy indicates that model performance does not improve at higher orders. Thus, low-order models might fit the data equally well.

Note When you use the same data set for estimation and validation, use the MDL and AIC criteria to select model orders. These criteria compensate for overfitting that results from using too many parameters.

Model Order for the First Input-Output Combination

In this tutorial, there are two inputs to the system and one output and you estimate model orders for each input/output combination independently. You can either estimate the delays from the two inputs simultaneously or one input at a time.

It makes sense to try the following order combinations for the first input/output combination:

- n_a=`2:5`
- n_b=`1:5`
- n_k=`5`

This is because the nonparametric models you estimated in “Estimating Nonparametric Models” on page 5-17 show that the response for the first input/output combination might have a second-order response. Similarly, in “Estimating Delays in the System” on page 5-22, the delay for this input/output combination was estimated to be 5.

To estimate model order for the first input/output combination:

1 Use `struc` to create a matrix of possible model orders.

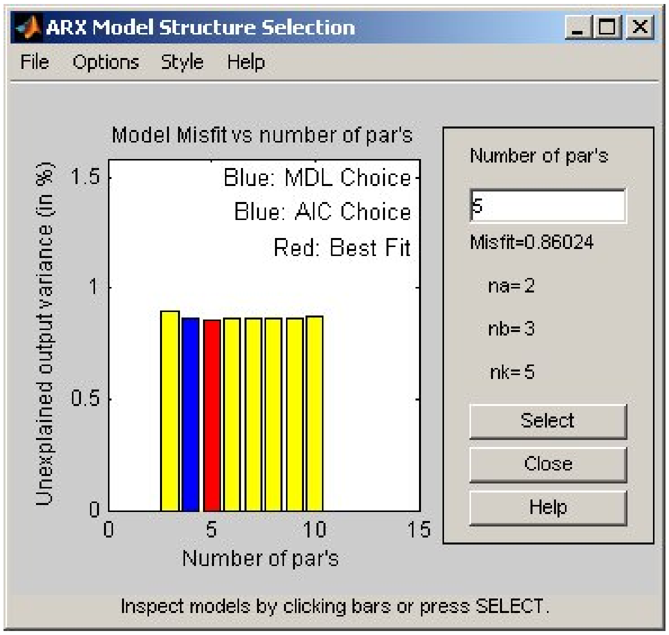
   ```
   NN1 = struc(2:5,1:5,5);
   ```

2 Use `selstruc` to compute the loss functions for the ARX models with the orders in `NN1`.

   ```
   selstruc(arxstruc(ze(:,:,1),zv(:,:,1),NN1))
   ```

   Note `(ze(:,:,1)` selects the first input in the data.

This command opens the interactive ARX Model Structure Selection window.



Misfit=0.86024

na=2

nb=3

nk=5

Inspect models by clicking bars or press SELECT.

Note The Rissanen MDL and Akaike AIC criteria produces equivalent results and are both indicated by a blue rectangle on the plot.

The red rectangle represents the best overall fit, which occurs for n_a=2, n_b=3, and n_k=5. The height difference between the red and blue rectangles is insignificant. Therefore, you can choose the parameter combination that corresponds to the lowest model order.

3 Click the blue rectangle, and then click **Select** to choose that combination of orders:

n_a=2

n_b=2

n_k=5

4 To continue, press any key while in the MATLAB Command Window.

Model Order for the Second Input-Output Combination

It makes sense to try the following order combinations for the second input/output combination:

- n_a=`1:3`
- n_b=`1:3`
- n_k=`10`

This is because the nonparametric models you estimated in “Estimating Nonparametric Models” on page 5-17 show that the response for the second input/output combination might have a first-order response. Similarly, in “Estimating Delays in the System” on page 5-22, the delay for this input/output combination was estimated to be 10.

To estimate model order for the second input/output combination:

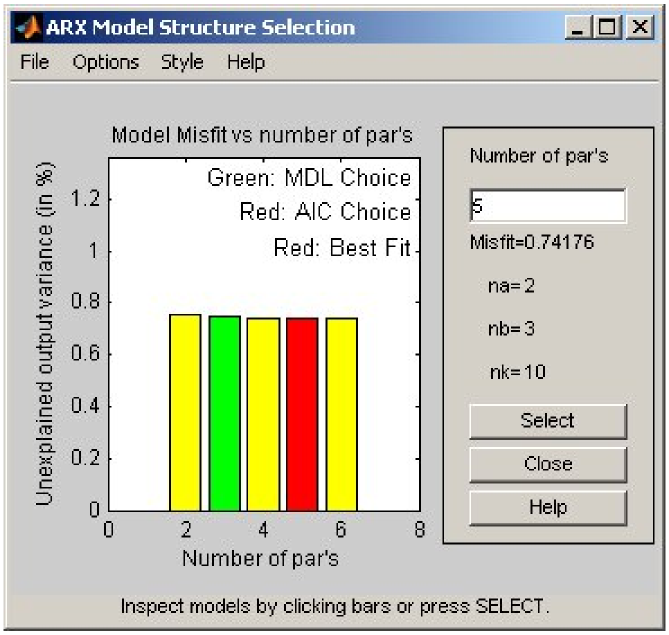
1 Use `struc` to create a matrix of possible model orders.

```
NN2 = struc(1:3,1:3,10);
```

2 Use `selstruc` to compute the loss functions for the ARX models with the orders in `NN2`.

```
selstruc(arxstruc(ze(:,:,2),zv(:,:,2),NN2))
```

This command opens the interactive ARX Model Structure Selection window.


ARX Model Structure Selection
File
Options
Style
Help
Model Misfit vs number of par's
Green: MDL Choice
Red: AIC Choice
Red: Best Fit
Unexplained output variance (in %)
Number of par's
Number of par's
5
Misfit=0.74176
na=2
nb=3
nk=10
Select
Close
Help
Inspect models by clicking bars or press SELECT.

Note The Akaike AIC and the overall best fit criteria produces equivalent results. Both are indicated by the same red rectangle on the plot.

The height difference between all the rectangles is insignificant, Thus, all combinations of orders produce similar model performance. Therefore, you can choose the parameter combination that corresponds to the lowest model order.

3 Click the yellow rectangle on the far left, and then click **Select** to choose the lowest order: n_a=1, n_b=1, and n_k=10.

4 To continue, press any key while in the MATLAB Command Window.

Estimating Continuous-Time Process Models

In this section...

Specifying the Structure of the Process Model

In this portion of the tutorial, you estimate a linear, continuous-time process model. System Identification Toolbox supports continuous-time process models with at most three poles (which might contain underdamped poles), one zero, a delay element, and an integrator.

You must have already prepared your data, as described in "Preparing Data" on page 5-5.

You can use the following results of estimated model orders to specify the orders of the process model:

- For the first input/output combination, use:
  - Two poles, corresponding to n_a=2 in the ARX model.
  - Delay of 5, corresponding to n_k=5 samples (or 2.5 minutes) in the ARX model.
- For the second input/output combination, use:
  - One pole, corresponding to n_a=1 in the ARX model.
  - Delay of 10, corresponding to n_k=10 samples (or 5 minutes) in the ARX model.

Note Because there is no relationship between the number of zeros estimated by the discrete-time ARX model and its continuous-time counterpart, you do not have an estimate for the number of zeros. In this tutorial, you can specify one zero for the first input/output combination, and no zero for the second-output combination.

Use the `idproc` command to create two process model structures, one for each input/output combination:

```
midproc0 = idproc({'P2ZUD','P1D'});
```

The argument of `idproc` is a cell array that contains two strings, where each string specifies the model structure for each input/output combination:

- `'P2ZUD'` represents a transfer function with two poles (`P2`), one zero (`Z`), underdamped (complex-conjugate) poles (`U`) and a delay (`D`).
- `'P1D'` represents a transfer function with one pole (`P1`) and a delay (`D`).

Viewing the Model Structure and Parameter Values

To view the two resulting process-model structures, type the following command at the MATLAB prompt:

```
midproc0
```

MATLAB responds with the following output:

```
Process model with 2 inputs: y = G_1(s)u_1 + G_2(s)u_2

where
                       1+Tz*s
G_1(s) = K * ---------------------- * exp(-Td*s)
              1+2*Zeta*Tw*s+(Tw*s)^2


with    K = NaN
       Tw = NaN
     Zeta = NaN
       Td = NaN
       Tz = NaN


           K
G_2(s) = ---------- * exp(-Td*s)
          1+Tp1*s

with   K = NaN
     Tp1 = NaN
      Td = NaN

This model was not estimated from data.
```

The parameter values are set to `NaN` because they are not yet estimated.

Specifying Initial Guesses for Time Delays

Set the time delay property of the model object using dot notation to `2.5` min and `5` min for both input/output combinations as initial guesses:

```
midproc0.Td = [2.5 5];
```

Note When setting the `Td` model property, you must specify the delays in terms of actual time units (minutes, in this case) and not the number of samples.

Estimating Model Parameters Using pem

pem is an *iterative* estimation method, which means that it uses an iterative nonlinear least-squares algorithm to minimize a cost function. The *cost function* is the weighted sum of the squares of the errors.

Depending on its arguments, pem estimates different black-box polynomial models. You can use pem, for example, to estimate parameters for linear continuous-time process, state-space, ARX, ARMAX, Box-Jenkins, and output-error model structures.

You must have already defined the process model structure, as described in "Specifying the Structure of the Process Model" on page 5-33.

To use pem, you must provide a model structure with unknown parameters and the estimation data as input arguments. In this case, use midproc0 as the model structure and Ze1 as the estimation data:

```
midproc = pem(Ze1,midproc0);
present(midproc)
```

MATLAB responds with the following estimated parameters:

```
Process model with 2 inputs: y = G_1(s)u_1 + G_2(s)u_2

where
                     1+Tz*s
G_1(s) = K * ---------------------- * exp(-Td*s)
             1+2*Zeta*Tw*s+(Tw*s)^2

with    K = 0.12845
       Tw = 0.70079
     Zeta = 17.876
       Td = 2.4739
       Tz = 477.14


           K
G_2(s) = ---------- * exp(-Td*s)
          1+Tp1*s

with   K = 10.418
     Tp1 = 2.1116
      Td = 4.8864

Estimated using PEM from data set Ze1
Loss function 6.2021 and FPE 6.30214
```

Unlike discrete-time polynomial models, continuous-time process models let you estimate the delays. In this case, the estimated delay values `2.4739` and `4.8864` are different from the initial values `2.5` and `5`, respectively.

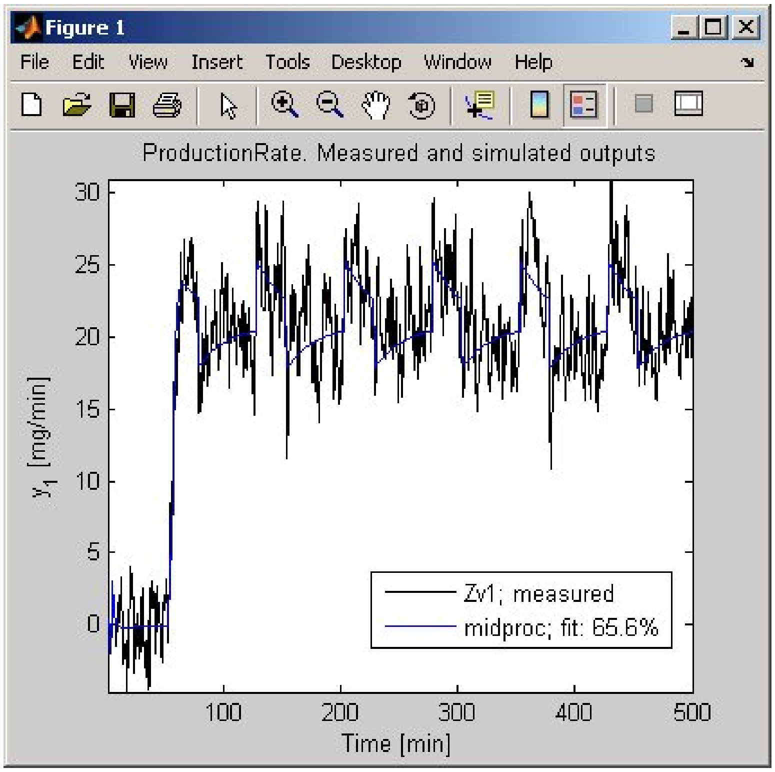
Learn More

To learn more about estimating process models, see the corresponding section in the System Identification Toolbox documentation.

Validating the Process Model

In this section, you create a plot that compares the actual output and the model output using the `compare` command:

```
compare(Zv1,midproc)
```

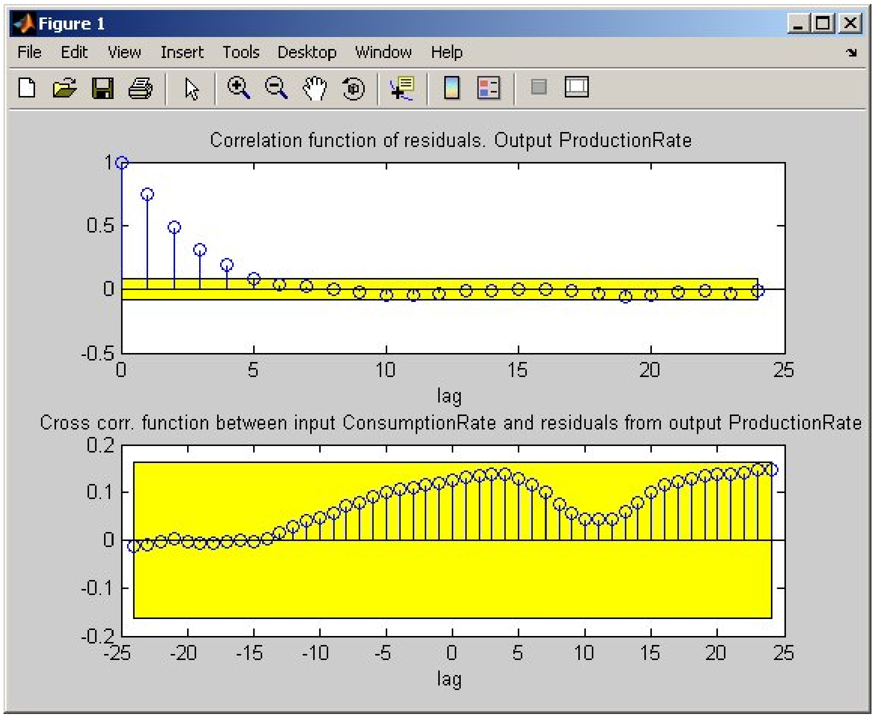


The preceding plot shows that the model output reasonably agrees with the measured output: there is an agreement of 65.6% between the model and the validation data.

Use `resid` to perform residual analysis:

```
resid(Zv1,midproc0)
```

Because the sample system has two inputs, there are two cross-correlation plots of the residuals with each input, as shown in the following figure.



Autocorrelation and Cross-Correlations of Residuals with the First Input

After MATLAB displays the first plot, press **Enter** to view the cross-correlation with the second input, as shown in the following figure.

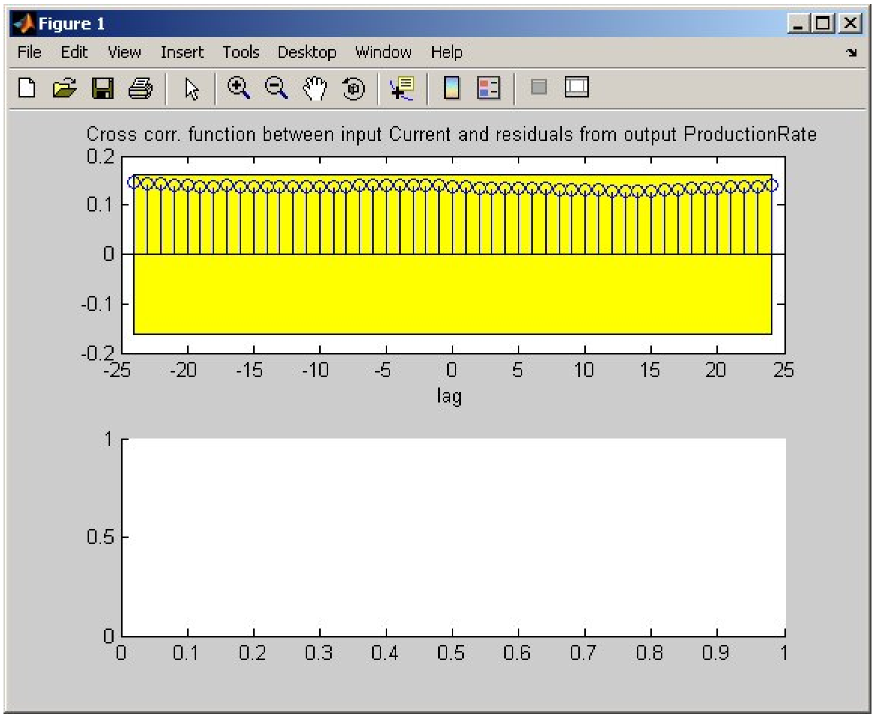


Figure 1
File Edit View Insert Tools Desktop Window Help
Cross corr. function between input Current and residuals from output ProductionRate
0.2
0.1
0
-0.1
-0.2
-25 -20 -15 -10 -5 0 5 10 15 20 25
lag
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Cross-Correlations of Residuals with the Second Input

In the preceding figure, the autocorrelation plot shows values outside the confidence region and indicates that the residuals are correlated. However, the cross-correlation with each of the two inputs shows no significant correlation. This lack of correlation indicates that this process model is accurate, but that there might be a need for a noise model.

Estimating a Noise Model to Improve Results

This portion of the tutorial shows how you can improve the process model by including a noise model.

In “Validating the Process Model” on page 5-38, you noticed that the process model performed well except that it produced correlated residuals. This correlation of residuals indicates evidence of unmodeled dynamics, which might be entering the system as an external disturbance.

Use the following command to specify a first-order ARMA noise:

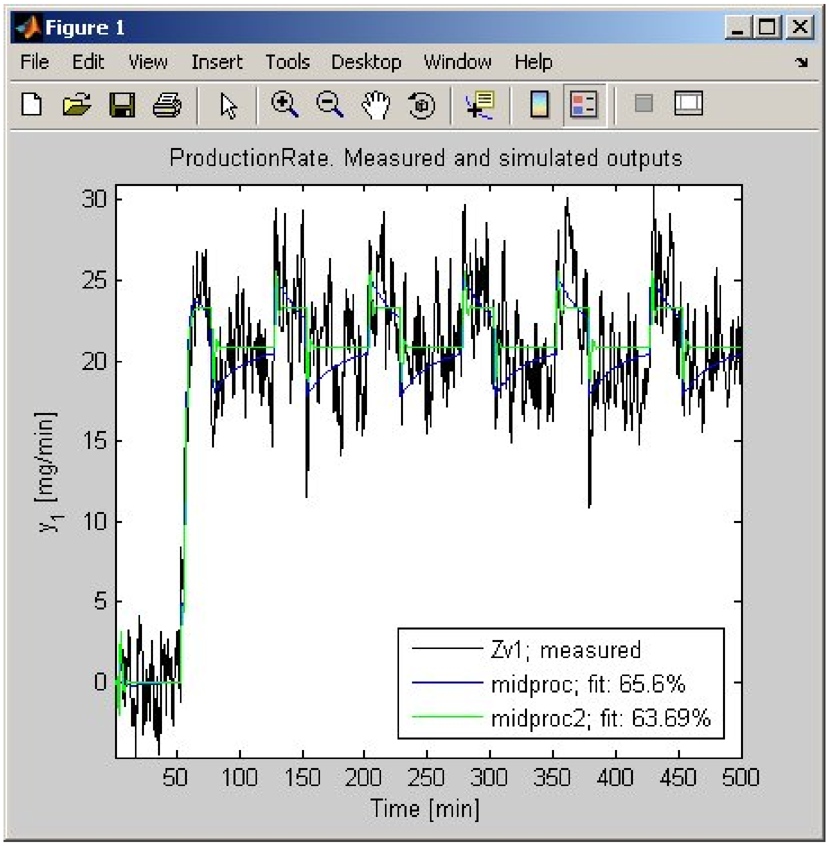
```
midproc2 = pem(Ze1,midproc0,'DisturbanceModel','arma1')
```

Note You can type `'dist'` instead of `'DisturbanceModel'`. Property names are not case sensitive, and you only need to include the portion of the name that uniquely identifies the property.

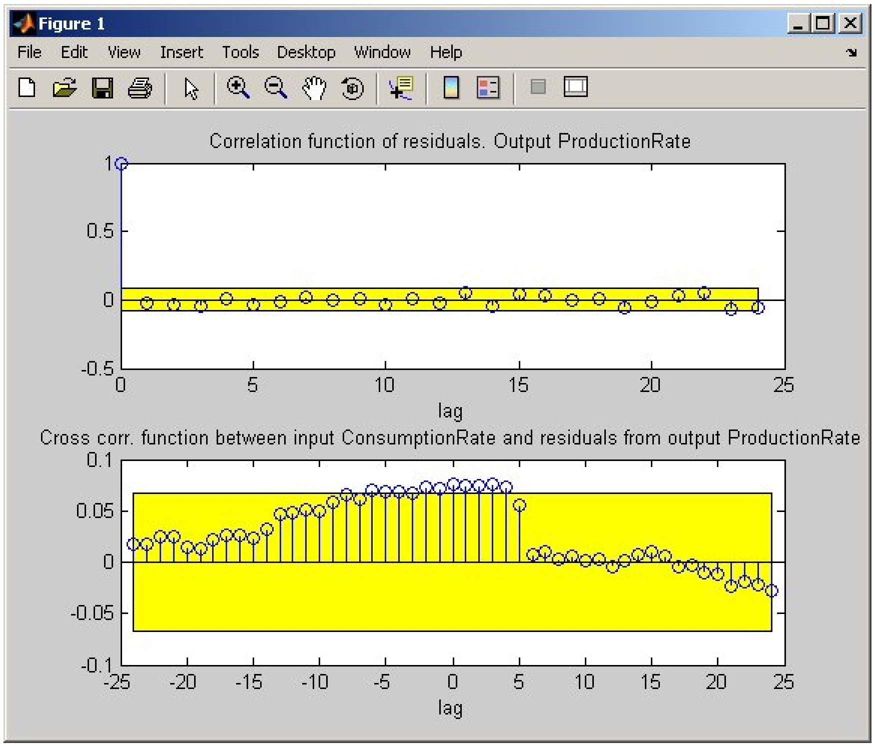
Compare the new model to the old model and to the measured data, and perform residual analysis, as follows:

```
compare(Zv1,midproc,midproc2)
figure
resid(Zv1,midproc2)
```

The following plot shows that the model output maintains reasonable agreement with the validation-data output. Press **Enter** to view the cross-correlation of the residuals with the second input.



However, the next plot shows that adding a noise model improves the result by producing uncorrelated residuals: the top set of axes show that the autocorrelation values are inside the confidence bounds.



Learn More

To learn more about estimating noise models, see “Estimating Noise Models” on page 2-17.

Estimating Black-Box Polynomial Models

In this section...

Initial Orders for Estimating Polynomial Models

In this portion of the tutorial, you estimate several different types of black-box polynomial models.

You must have already prepared your data, as described in "Preparing Data" on page 5-5.

You can use the following results of estimated model orders to specify the orders of the process model:

- For the first input/output combination, use:
  - Two poles, corresponding to n_a=2 in the ARX model.
  - One zero, corresponding to n_b=2 in the ARX model.
  - Delay of 5, corresponding to n_k=5 samples (or 2.5 minutes) in the ARX model.
- For the second input/output combination, use:
  - One pole, corresponding to n_a=1 in the ARX model.
  - No zeros, corresponding to n_b=1 in the ARX model.
  - Delay of 10, corresponding to n_k=10 samples (or 5 minutes) in the ARX model.

Estimating a Linear ARX Model

About ARX Models

For a single-input/single-output system (SISO), the ARX model structure is:

$$\begin{aligned} &y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-n_a) = \\ &\qquad \mathrm{b}_1 u(t-n_k) + \ldots + b_{nb} u(t-n_k-n_b+1) + e(t) \end{aligned}$$

y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, n_a is the number of poles, n_b is the number of zeros plus 1, n_k is the number of samples before the input affects the system output, and *e(t)* is the white-noise disturbance.

The ARX model structure does not distinguish between the poles for individual input/output paths: dividing the ARX equation by *A*, which contains the poles, shows that *A* appears in the denominator for both inputs. Therefore, you can set n_a to the sum of the poles for each input/output pair, which is equal to `3` in this case.

You must specify the model orders to estimate ARX models.

System Identification Toolbox estimates the parameters $a_1 \ldots a_n$ and $b_1 \ldots b_n$ using the data and the model orders you specify.

Estimating ARX Models Using arx

Use `arx` to compute the polynomial coefficients using a fast, noniterative method:

```
marx = arx(Ze1,'na',3,'nb',[2 1],'nk',[5 10]);
present(marx) % Displays model parameters
```

MATLAB estimates the polynomials A, B1, and B2:

```
Discrete-time IDPOLY model: A(q)y(t) = B(q)u(t) + e(t)
A(q) = 1 - 1.027 (+-0.02917) q^-1
         + 0.1675 (+-0.04214) q^-2
         + 0.01307 (+-0.02591) q^-3
B1(q) = 1.86 (+-0.1896) q^-5 - 1.608 (+-0.1894) q^-6
B2(q) = 1.612 (+-0.07417) q^-10
```

The uncertainty for each of the model parameters is computed to 1 standard deviation and appears in parentheses next to each parameter value.

Tip Alternatively, you can use the following shorthand syntax and specify model orders as a single vector:

```
marx = arx(Ze1,[3 2 1 5 10])
```

Accessing Model Data

The model you estimated, marx, is a discrete-time idpoly object. To get the properties of this model object, you can use the get function:

```
get(marx)
```

MATLAB returns the following model properties:

```
                a: [1 -0.9861 0.1512 0.0095]
                b: [2x11 double]
                c: 1
                d: 1
                f: [2x1 double]
               da: [0 0.0301 0.0424 0.0261]
               db: [2x11 double]
               dc: 0
               dd: 0
               df: [2x1 double]
               na: 3
               nb: [2 1]
               nc: 0
               nd: 0
               nf: [0 0]
               nk: [5 10]
     InitialState: 'Auto'
             Name: ''
               Ts: 0.5000
        InputName: {2x1 cell}
        InputUnit: {2x1 cell}
       OutputName: {'ProductionRate'}
       OutputUnit: {'mg/min'}
         TimeUnit: 'min'
  ParameterVector: [6x1 double]
            PName: {}
 CovarianceMatrix: [6x6 double]
    NoiseVariance: 2.7732
       InputDelay: [2x1 double]
        Algorithm: [1x1 struct]
   EstimationInfo: [1x1 struct]
            Notes: {}
         UserData: []
```

You can access the information stored by these properties using dot notation. For example, you can compute the discrete poles of the model by finding the roots of the *A* polynomial:

```
marx_poles=roots(marx.a)
```

In this case, you access the *A* polynomial using `marx.a`.

MATLAB returns the following output:

```
marx_poles =

    0.7751
    0.2585
   -0.0475
```

Thus, the model `marx` describes system dynamics using three discrete poles.

Tip You can also use the `zpkdata` command to compute the poles of a model directly.

Learn More

To learn more about estimating polynomial models, see the corresponding sections in the *System Identification Toolbox User's Guide*.

For more information about accessing model data, see the topic on extracting numerical data from linear models in the *System Identification Toolbox User's Guide*.

Estimating a State-Space Model

About State-Space Models

The general state-space model structure is:

$$x(t+1) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

y(t) represents the output at time *t*, *u(t)* represents the input at time *t*, *x(t)* is the state values at time *t*, and *e(t)* is the white-noise disturbance.

You must specify a single integer as the model order to estimate a state-space model. By default, the delay equals `1`.

System Identification Toolbox estimates the state-space matrices *A*, *B*, *C*, *D*, and *K* using the model order and the data you specify.

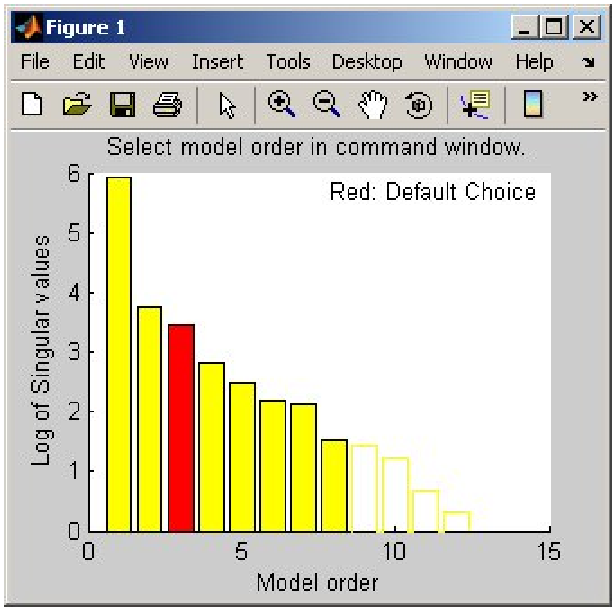
The state-space model structure is a good choice for quick estimation because it contains only two parameters: `n` is the number of poles (the size of the *A* matrix) and `nk` is the delay.

Estimating State-Space Models Using n4sid

Use the `n4sid` command to specify a range of model orders and evaluate the performance of several state-space models (orders 2 to 8):

```
mn4sid = n4sid(Ze1,2:8,'nk',[5 10]);
```

This command uses the fast, noniterative (subspace) method and opens the following plot. You use this plot to decide which states provide a significant relative contribution to the input/output behavior, and which states provide the smallest contribution.



The vertical axis is a relative measure of how much each state contributes to the input/output behavior of the model (*log of singular values of the covariance matrix*). The horizontal axis corresponds to the model order `n`. System Identification Toolbox recommends `n=3`, indicated by a red rectangle.

To select this model order, type `3` in the MATLAB Command Window, and press **Enter**.

By default, `n4sid` uses a free parameterization of the state-space form. To estimate a canonical form instead, set the value of the `SSParameterization` property to `'Canonical'`:

```
mCanonical = n4sid(Ze1,3,'nk',[5 10],...
             'ssparameterization','canonical');
present(mCanonical)    % Displays model properties
```

Note When you examine the displayed properties, notice that the model order is high. This high order occurs because the model uses additional states to incorporate the input delays.

Learn More

To learn more about estimating state-space models, see the corresponding section in the *System Identification Toolbox User's Guide*.

Estimating a Box-Jenkins Model

About Box-Jenkins Models

The general Box-Jenkins (BJ) structure is:

$$y(t) = \sum_{i=1}^{nu} \frac{B_i(q)}{F_i(q)} u_i\left(t - nk_i\right) + \frac{C(q)}{D(q)} e(t)$$

To estimate a BJ model, you need to specify the parameters n_b, n_f, n_c, n_d, and n_k.

Whereas the ARX model structure does not distinguish between the poles for individual input/output paths, the BJ model provides more flexibility in modeling the poles and zeros of the disturbance separately from the poles and zeros of the system dynamics.

Estimating a BJ Model Using pem

You can use to estimate the BJ model. pem is an iterative method and has the following general syntax:

```
pem(data,'na',na,'nb',nb,'nc',nc,'nd',nd,'nf',nf,'nk',nk)
```

In this case, `data` is an `iddata` or `idfrd` object, and `na`, `nb`, `nc`, `nd`, `nf`, and `nk` specify the model order.

To estimate the BJ model using `pem`, type the following command at the MATLAB prompt:

```
mbj = pem(Ze1,'nf',[2 1],'nb',[2 1],'nc',1,'nd',1,'nk',[5 10]);
present(mbj)
```

This command specifies `nf=2`, `nb=2`, `nk=5` for the first input, and `nf=nb=1` and `nk=10` for the second input

---

Tip Alternatively, you can use the following shorthand syntax that specifies the orders as a single vector:

```
mbj = bj(Ze1,[2 1 1 1 2 1 5 10]);
```

`bj` is a version of `pem` that specifically estimates the BJ model structure.

MATLAB estimates the polynomial coefficients, as follows:

```
Discrete-time IDPOLY model:
y(t) = [B(q)/F(q)]u(t) + [C(q)/D(q)]e(t)
B1(q) = 1.903 (+-0.1888) q^-5 - 1.469 (+-0.2304) q^-6
B2(q) = 2.086 (+-0.09506) q^-10
C(q) = 1 + 0.1149 (+-0.04131) q^-1
D(q) = 1 - 0.7364 (+-0.02856) q^-1
F1(q) = 1 - 1.361 (+-0.06205) q^-1 + 0.5982 (+-0.05408) q^-2
F2(q) = 1 - 0.8031 (+-0.009455) q^-1
```

The uncertainty for each of the model parameters is computed to 1 standard deviation and appears in parentheses next to each parameter value.

The polynomials *C* and *D* give the numerator and the denominator of the noise model, respectively.

Learn More

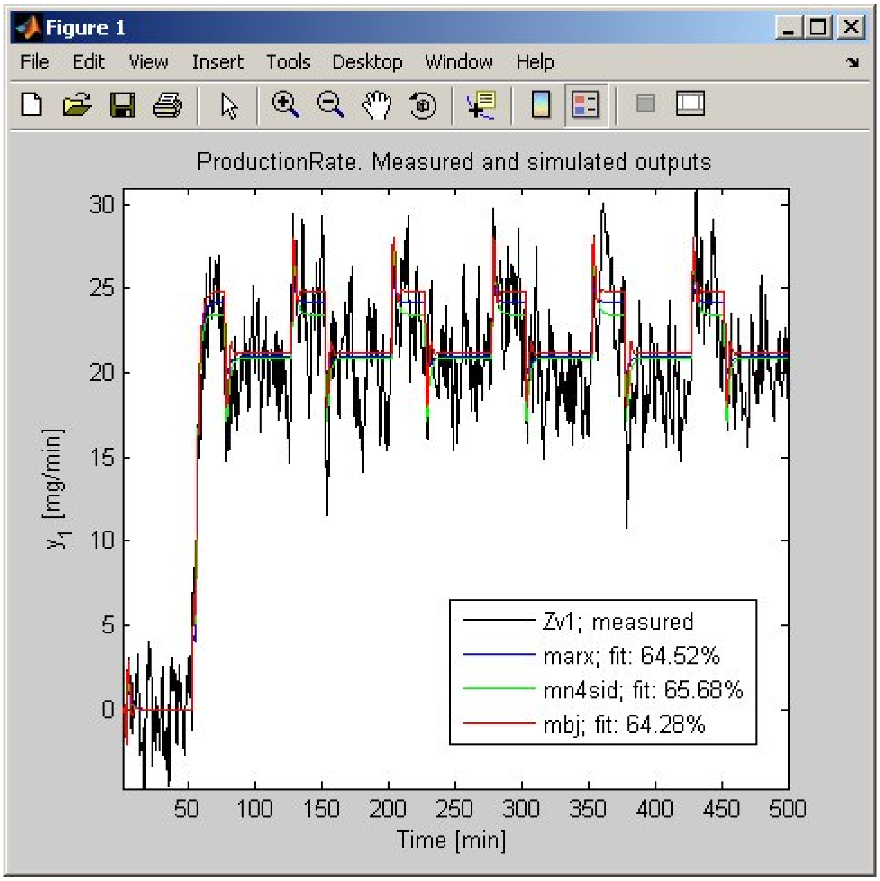
To learn more about estimating state-space models, see the corresponding section in the *System Identification Toolbox User's Guide*.

Comparing Models

To compare the output of the ARX, state-space, and Box-Jenkins models with the measured output, use the `compare` function:

```
compare(Zv1,marx,mn4sid,mbj)
```

`compare` plots the measured output in the validation data set against the simulated output from the models. The input data from the validation data set serves as input to the models.



Measured Output and Simulated Outputs

To perform residual analysis on the ARX model, type the following command:

```
resid(Zv1,marx)
```

Press **Enter** to view the cross-correlation with the second input. Because the sample system has two inputs, there are two plots showing the cross-correlation of the residuals with each input.

To perform residual analysis on the state-space model, type the following command:

```
resid(Zv1,mn4sid)
```

Finally, to perform residual analysis on the BJ model, type the following command:

```
resid(Zv1,mbj)
```

All three models simulate the output equally well and have uncorrelated residuals. Therefore, choose the ARX model because it is the simplest of the three black-box parametric models and adequately captures the process dynamics.

Simulating and Predicting Model Output

In this section...

Simulating the Model Output

In this portion of the tutorial, you simulate the model output. You must have already created the continuous-time process model `midproc2`, as described in "Estimating Continuous-Time Process Models" on page 5-33.

Simulating the model output requires the following information:

- Input values to the model
- Initial conditions for the simulation (also called *initial states*)

For example, the following commands use the `iddata` and `idinput` commands to construct an input data set, and use `sim` to simulate the model output:

```
% Create input for simulation
U = iddata([],idinput([200 2]),'Ts',0.5);
% Simulate the response setting initial conditions
% equal to zero
ysim_1 = sim(midproc2,U,'InitialState','zero')
```

Carefully consider which initial conditions you use in the simulation. A system produces different responses for different initial conditions, even when the input data to the model is the same. If you do not specify the correct initial states, your model response does not match the measured output for your data. Therefore, when you use simulation to validate a model by matching simulated response to measured response, you must estimate the initial conditions from the measured data and use these as the initial states of the simulation.

Use `pe` to estimate the initial conditions `X0e` in the data set `Zv1`:

```
[Err,X0e] = pe(midproc2,Zv1,'estimate');
```

This function also computes the prediction errors `Err` between the simulated and the measured outputs.

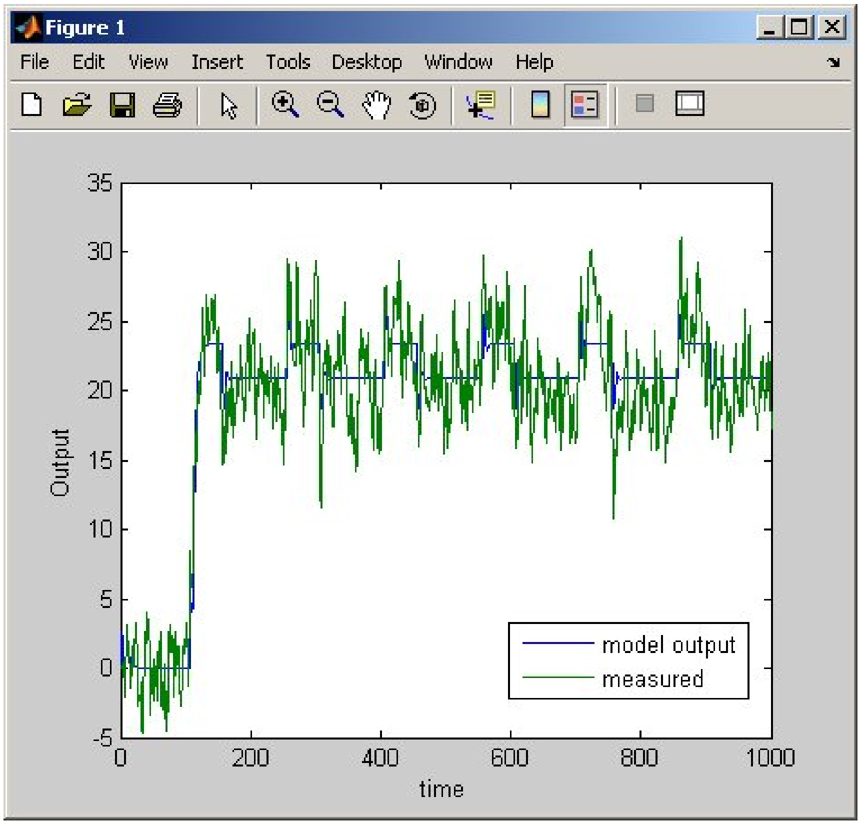
Next, simulate the model using the initial states estimated from the data:

```
ysim_2 = sim(midproc2,U,'InitialState',X0e);
```

Compare the simulated and the measured output on a plot:

```
figure
plot([ysim_2, Zv1.y])
legend({'model output','measured'})
xlabel('time'), ylabel('Output')
```

The comparison of simulated and measured output is displayed in the following figure.



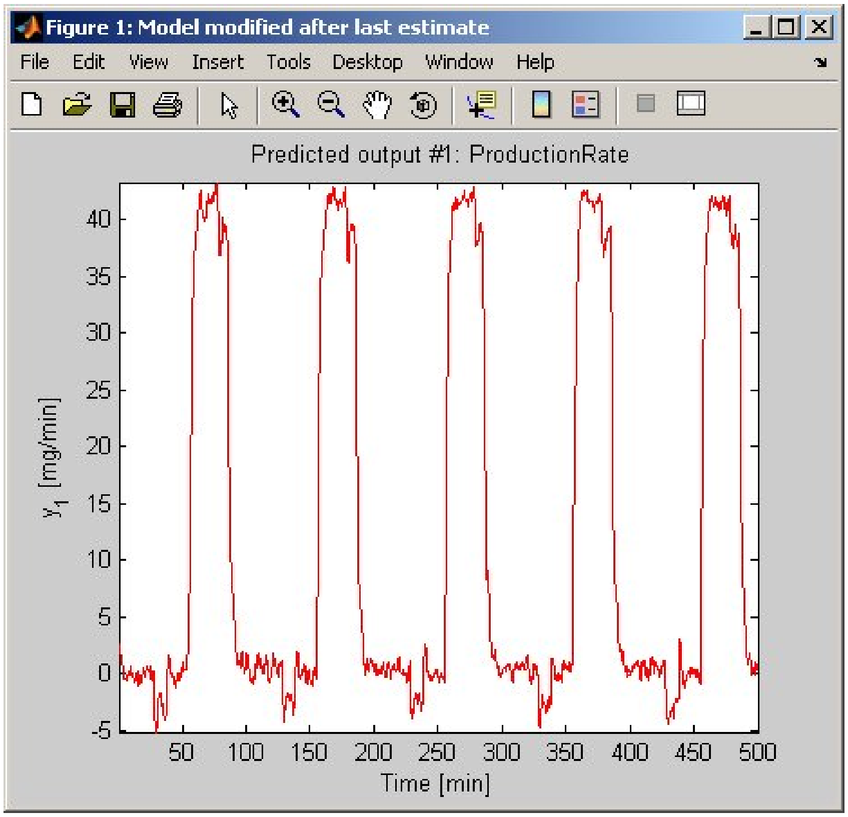
Predicting the Future Output

Many control-design applications require you to predict the future outputs of a dynamic system using the past input/output data.

For example, use `predict` to predict the model response five steps ahead:

```
predict(midproc2,Ze1,5)
```

The predicted output is displayed in the following figure.

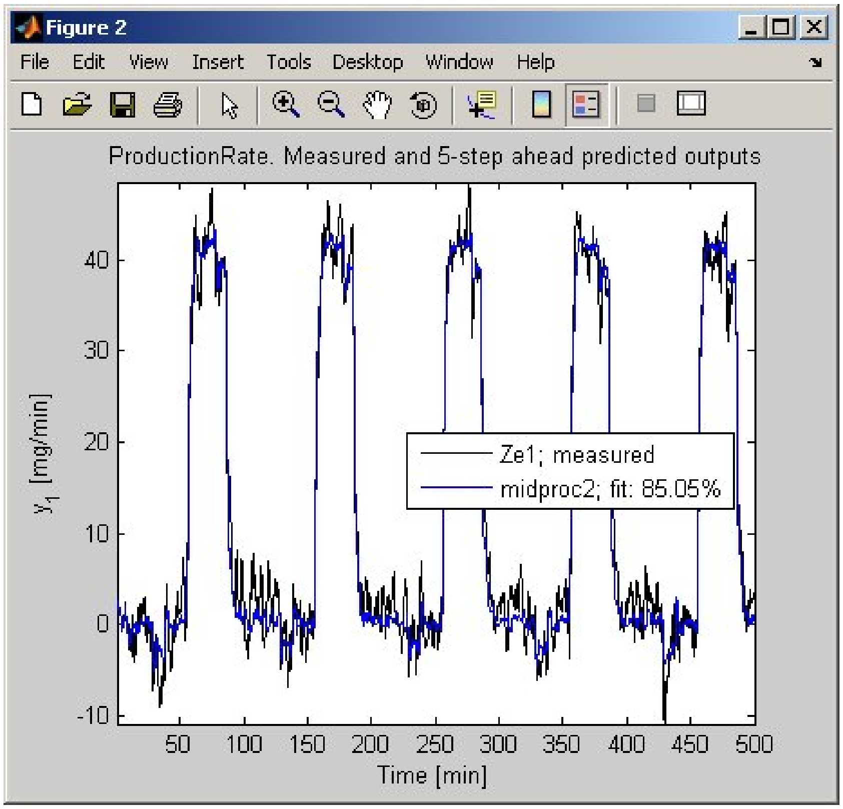


To compare the predicted output values with the measured output values, use the following command:

```
compare(Ze1,midproc2,5)
```

The third argument of `compare` specifies a five-step-ahead prediction, as shown in the following figure.

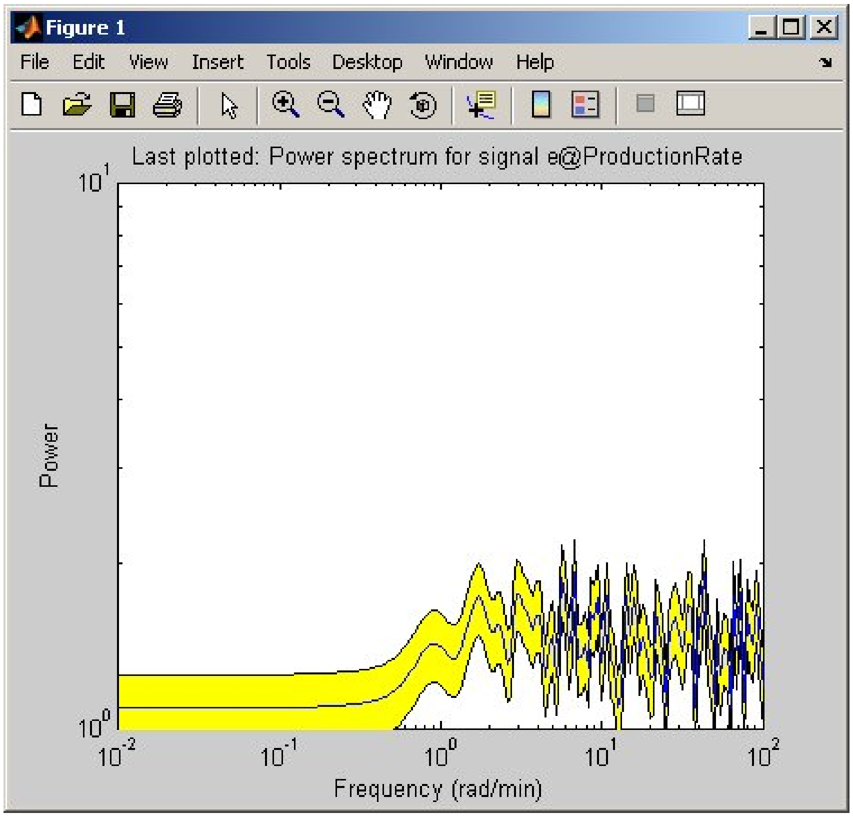
Note When you do not specify a third argument, as in “Simulating the Model Output” on page 5-56, `compare` assumes an infinite prediction horizon and simulates the model output.



Use pe to compute the prediction error Err between the predicted output of midproc2 and the measured output. Then, plot the error spectrum on a Bode plot.

```
[Err] = pe(midproc2,Zv1);
bode(spa(Err,[],logspace(-2,2,200)),...
            'mode','same','sd',1,'fill')
```

As shown in the following figure, the prediction errors are plotted with a 1-standard-deviation confidence interval. The errors are greater at high frequencies because of the high-frequency nature of the disturbance.



Tutorial: Estimating Nonlinear Black-Box Models

About This Tutorial

In this section...

Objectives

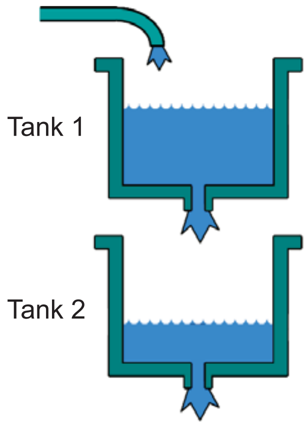
Estimate and validate nonlinear models from single-input/single-output (SISO) data to find the one that best represents your system dynamics.

After completing this tutorial, you will be able to accomplish the following tasks using the System Identification Tool GUI:

- Import data objects from the MATLAB Workspace browser into the GUI.
- Estimate and validate nonlinear models from the data.
- Plot and analyze the behavior of the nonlinearities.

Sample Data

The sample data you use in this tutorial is in `twotankdata.mat`, which contains SISO time-domain data for a two-tank system, shown in the following figure.



Two-Tank System

In the two-tank system, water pours through a pipe into Tank 1, drains into Tank 2, and leaves the system through a small hole at the bottom of Tank 2. The measured input *u(t)* to the system is the voltage applied to the pump that feeds the water into Tank 1 (in volts). The measured output *y(t)* is the height of the water in the lower tank (in meters).

Based on Bernoulli's law, which states that water flowing through a small hole at the bottom of a tank depends nonlinearly on the level of the water in the tank, you expect the relationship between the input and the output data to be nonlinear.

`twotankdata.mat` includes 3000 samples with a sampling interval of 0.2 s.

What Are Nonlinear Black-Box Models?

In this section...

Types of Nonlinear Black-Box Models

System Identification Toolbox lets you estimate nonlinear discrete-time black-box models for both single-output and multiple-output time-domain data. You can choose from two types of nonlinear, black-box model structures:

- Nonlinear ARX models
- Hammerstein-Wiener models

Note You can estimate Hammerstein-Wiener black-box models from input/output data only. These models do not support time-series data, where there is no input.

To learn how to estimate nonlinear black-box models at the command line, see the topics on estimating nonlinear models in *System Identification Toolbox User's Guide*.

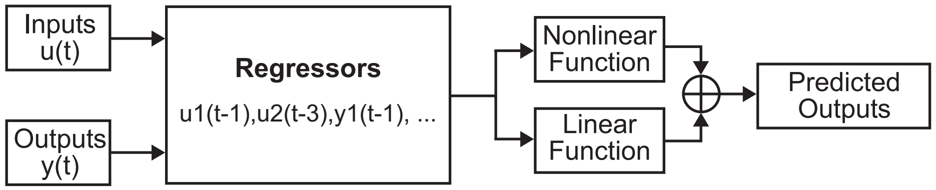
What Is a Nonlinear ARX Model?

Nonlinear ARX models describe nonlinear structures using a parallel combination of nonlinear and linear blocks. The nonlinear and linear functions are expressed in terms of variables called regressors, which System Identification Toolbox calculates using the models orders you specify.

The model output is a function of the *regressors*, such that:

$$\hat{y} = g\left(y(t-1), u(t-1), y(t-2), \ldots\right)$$

The function *g* is a combination of a linear function and a nonlinear function. The nonlinear function might be a binary partition tree, a neural network, or a network based on wavelets. The following figure shows how the predicted output of the model is formed from the inputs and outputs.

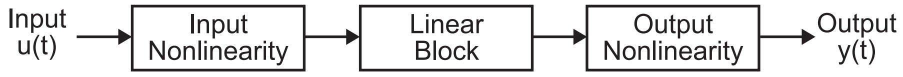


System Identification Toolbox computes regressors by performing transformations of the measured input *u(t)* and output *y(t)* signals. For example, regressors can be delayed inputs and outputs, such as *u(t-1)* and *y(t-3)*. Regressors can also be nonlinear functions of inputs and outputs, such as *tan(u(t-1))* or *u(t-1)y(t-3)*. You can either use default regressors, or specify your own custom functions of input and output signals.

You choose a nonlinear structure that independently combines linear and nonlinear regressors and the structure of the nonlinearity itself, such as a binary partition tree or a network based on wavelets. System Identification Toolbox uses input/output data to find the linear and nonlinear mappings that give the best predicted outputs of the nonlinear model.

What Is a Hammerstein-Wiener Model?

Hammerstein-Wiener models describe nonlinear structures using one or two static nonlinear blocks (no dynamics) in series with a linear block. Specifically, the input signal comes into a static nonlinearity, then goes into a linear dynamic system, and finally passes into a second static nonlinearity, as shown in the following figure.



In System Identification Toolbox, the linear block is a discrete-time transfer function and the nonlinear blocks are implemented using nonlinearity estimators. If you know that your system includes saturation or dead-zone nonlinearities, you can specify these specialized nonlinearity estimators in your model.

Preparing Data

In this section...

Loading Data into the MATLAB Workspace Browser

Load sample data in `twotankdata.mat` by typing the following command at the MATLAB prompt:

```
load twotankdata
```

This command loads the following two variables into the MATLAB Workspace browser:

- `y` is the output data, which is the water height in Tank 2 (in meters).
- `u` is the input data, which is the voltage applied to the pump that feeds the water into Tank 1 (in volts).

Creating iddata Objects

System Identification Toolbox data objects encapsulate both data values and data properties into a single entity. System Identification Toolbox commands let you conveniently manipulate these data objects as single entities.

You must have already loaded the sample data into the MATLAB Workspace browser, as described in "Loading Data into the MATLAB Workspace Browser" on page 6-8.

Use the following commands to create two data objects, `ze` and `zv`, where `ze` contains data for model estimation and `zv` contains data for model validation. `Ts` is the sampling interval.

```
Ts = 0.2; % Sampling interval is 0.5 min
z = iddata(y,u,Ts);
% First 1000 samples used for estimation
ze = z(1:1000);
% Remaining samples used for validation
zv = z(1001:3000);
```

To view the properties of an `iddata` object, use the `get` command. For example, type this command to get the properties of the estimation data:

```
get(ze)
```

MATLAB returns the following data properties and values:

```
              Domain: 'Time'
                Name: []
          OutputData: [1000x1 double]
                   y: 'Same as OutputData'
          OutputName: {'y1'}
          OutputUnit: {''}
           InputData: [1000x1 double]
                   u: 'Same as InputData'
           InputName: {'u1'}
           InputUnit: {''}
              Period: Inf
         InterSample: 'zoh'
                  Ts: 0.2000
              Tstart: 0.2000
    SamplingInstants: [1000x0 double]
            TimeUnit: ''
      ExperimentName: 'Exp1'
               Notes: []
            UserData: []
```

To learn more about these properties, see the `iddata` reference pages.

To modify data properties, you can use dot notation or the `set` command. For example, to assign channel names and units that label plot axes, type the following syntax at the MATLAB prompt:

```
% Set time units to minutes
ze.TimeUnit = 'sec';
% Set names of input channels
ze.InputName = 'Voltage';
% Set units for input variables
ze.InputUnit = 'V';
% Set name of output channel
ze.OutputName = 'Height';
% Set unit of output channel
ze.OutputUnit = 'm';

% Set validation data properties
zv.TimeUnit = 'sec';
zv.InputName = 'Voltage';
zv.InputUnit = 'V';
zv.OutputName = 'Height';
zv.OutputUnit = 'm';
```

To verify that the `InputName` property of `ze` is changed, type the following command:

```
ze.inputname
```

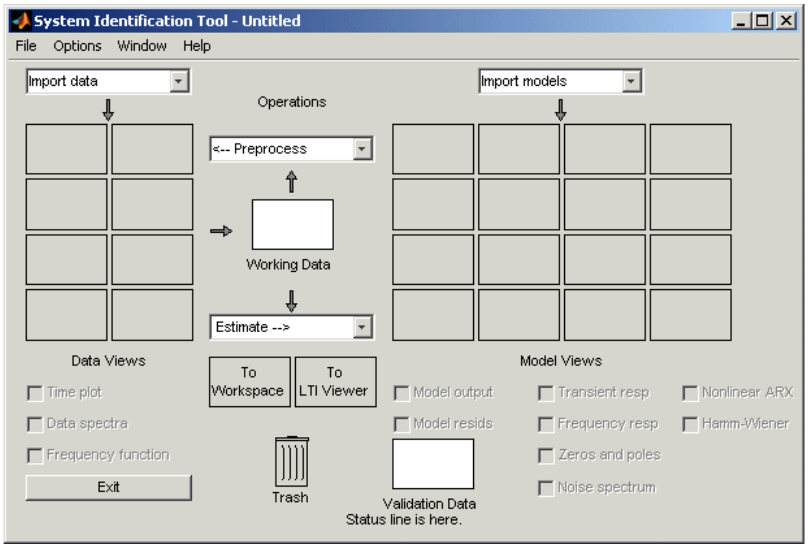
Tip Property names, such as `InputName`, are not case sensitive. You can also abbreviate property names that start with `Input` or `Output` by substituting `u` for `Input` and `y` for `Output` in the property name. For example, `OutputUnit` is equivalent to `yunit`.

Starting the System Identification Tool

To open the System Identification Tool GUI, type the following command at the MATLAB prompt:

```
ident
```

The default session name, `Untitled`, displays in the title bar.

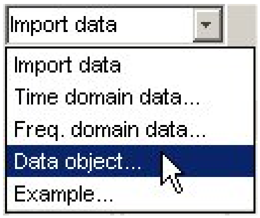


Importing Data Objects into the System Identification Tool

You can import the data objects into the GUI from the MATLAB Workspace browser.

You must have already created the data objects, as described in "Creating iddata Objects" on page 6-8, and opened the GUI, as described in "Starting the System Identification Tool" on page 6-10.

1 In the System Identification Tool window, select **Import data > Data object**. This action opens the Import Data dialog box.



2 Enter ze in the **Object** field to import the estimation data. Press **Enter**. This action enters the object information into the fields.

   Click **More** to view the following additional information about this data, including channel names and units.

3 Click **Import** to add the icon named ze to the System Identification Tool window.

4 In the Import Data dialog box, type zv in the **Object** field to import the validation data. Press **Enter**.

5 Click **Import** to add the icon named zv to the System Identification Tool window.

6 In the Import Data dialog box, click **Close**.

7 In the System Identification Tool window, drag the **ze** icon to the **Working Data** rectangle, and drag the **zv** icon to the **Validation Data** rectangle.

Estimating Nonlinear ARX Models

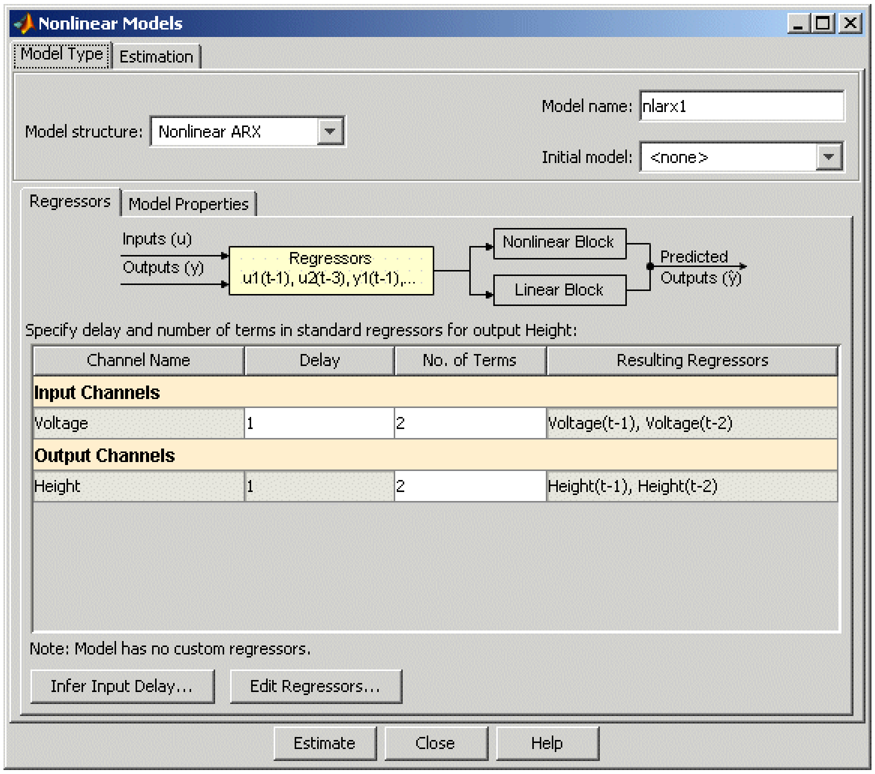
In this section...

Estimating a Nonlinear ARX Model with Default Settings

In this portion of the tutorial, you estimate a nonlinear ARX model using default estimation options.

You must have already prepared the data, as described in "Preparing Data" on page 6-8.

1 In the System Identification Tool window, select **Estimate > Nonlinear models** to open the Nonlinear Models dialog box.



Model structure: Nonlinear ARX

Model name: nlarx1

Initial model: <none>

Regressors | Model Properties

Specify delay and number of terms in standard regressors for output Height:

Channel Name	Delay	No. of Terms	Resulting Regressors
Input Channels			
Voltage	1	2	Voltage(t-1), Voltage(t-2)
Output Channels			
Height	1	2	Height(t-1), Height(t-2)

Note: Model has no custom regressors.

The **Model Type** tab is already open and the default **Model Structure** is `Nonlinear ARX`.

In the **Regressors** tab, the model orders for both **Input Channels** and **Output Channels** are specified by the **Delay** of `1` and **No. of Terms** equal to `2`. Thus, the model output *y(t)* is related to the input *u(t)* via the following nonlinear autoregressive equation:

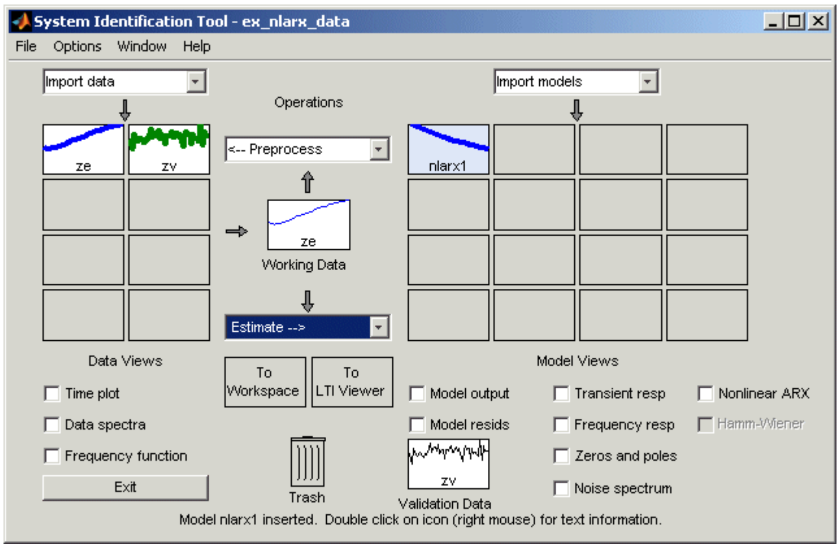
$$y(t) = f\left(y(t-1), y(t-2), u(t-1), u(t-2)\right)$$

f is the nonlinearity estimator you select in the **Model Properties** tab.

2 Click the **Model Properties** tab.

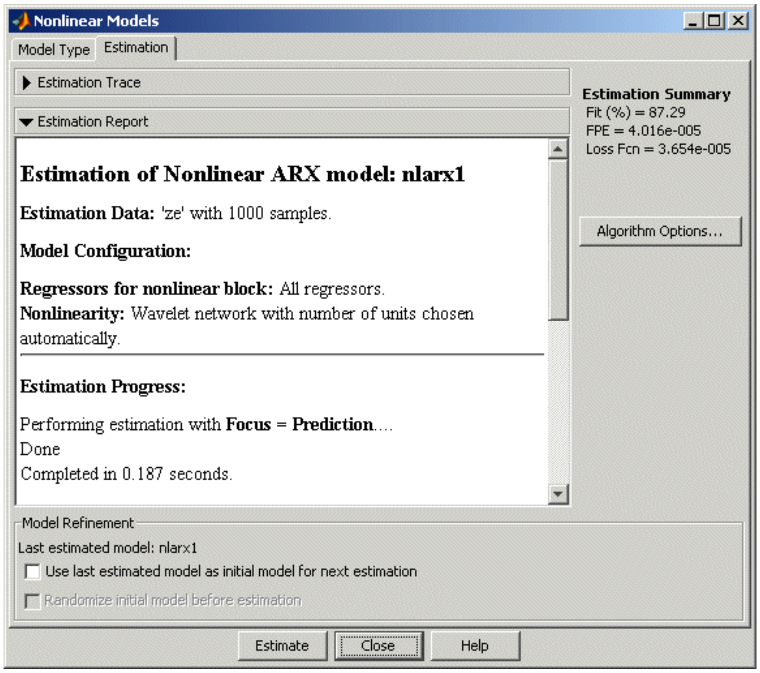
The **Nonlinearity** represents the nonlinear function *f* and is already set to `Wavelet Network`, by default. The number of units for the nonlinearity estimator is set to **Select automatically**, which lets the algorithm search for the best number of nonlinearity units during estimation.

3 Click **Estimate**. This selection adds the model `nlarx1` to the System Identification Tool window, as shown in the following figure.



Model nlarx1 inserted. Double click on icon (right mouse) for text information.

The Nonlinear Models dialog box displays the following estimation information in the **Estimation** tab.



Estimation Summary
Fit (%) = 87.29
FPE = 4.016e-005
Loss Fcn = 3.654e-005

Estimation of Nonlinear ARX model: nlarx1

Estimation Data: 'ze' with 1000 samples.

Model Configuration:

Regressors for nonlinear block: All regressors.
Nonlinearity: Wavelet network with number of units chosen automatically.

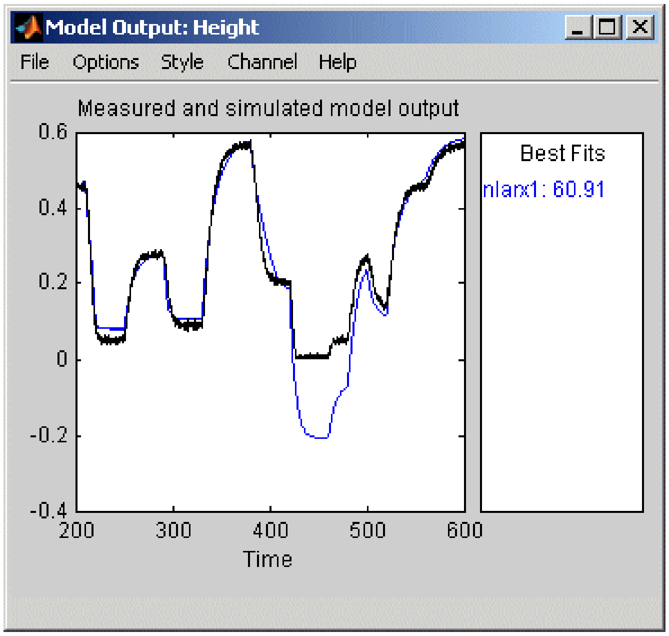
Estimation Progress:

Performing estimation with **Focus** = **Prediction**....
Done
Completed in 0.187 seconds.

Model Refinement
Last estimated model: nlarx1
☐ Use last estimated model as initial model for next estimation
☐ Randomize initial model before estimation

Note **Fit** (%) is computed using the estimation data set, and not the validation data set. However, the model output plot shows the fit to the validation data set.

4 In the System Identification Tool window, select the **Model output** check box. Simulation of the model output uses the input validation data as input to the model. It plots the simulated output on top of the output validation data.



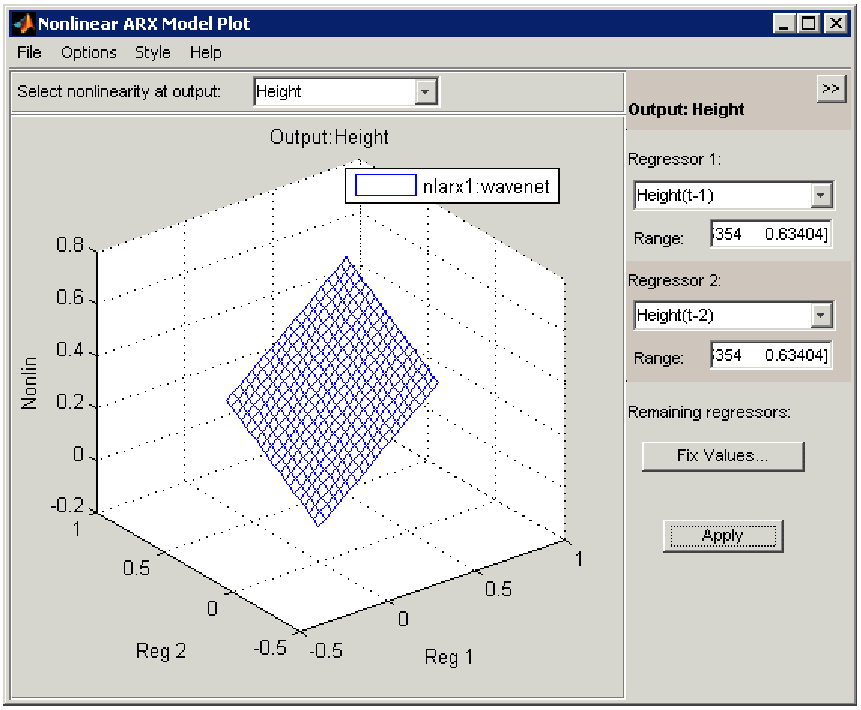
The **Best Fits** area of the Model Output plot shows that the agreement between the model output and the validation-data output is 60.91%.

Plotting Nonlinearity Cross-Sections for Nonlinear ARX Models

Perform the following procedure to view the shape of the nonlinearity as a function of regressors on a Nonlinear ARX Model plot.

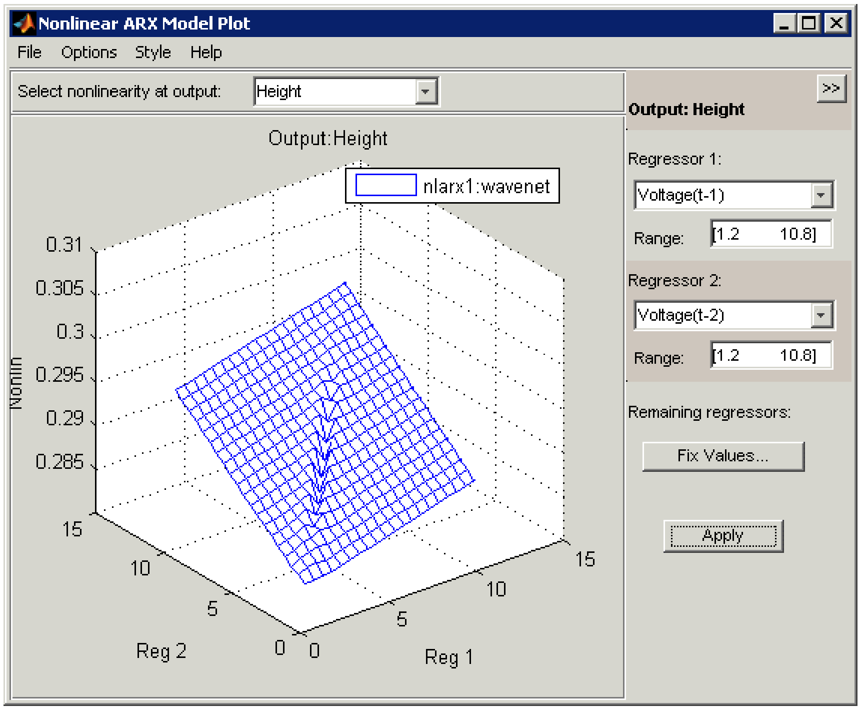
1 In the System Identification Tool window, select the **Nonlinear ARX** check box to view the nonlinearity cross-sections.

By default, the plot shows the relationship between the output regressors `Height(t-1)` and `Height(t-2)`. This plot shows a regular plane in the following figure. Thus, the relationship between the regressors is approximately a linear plane.



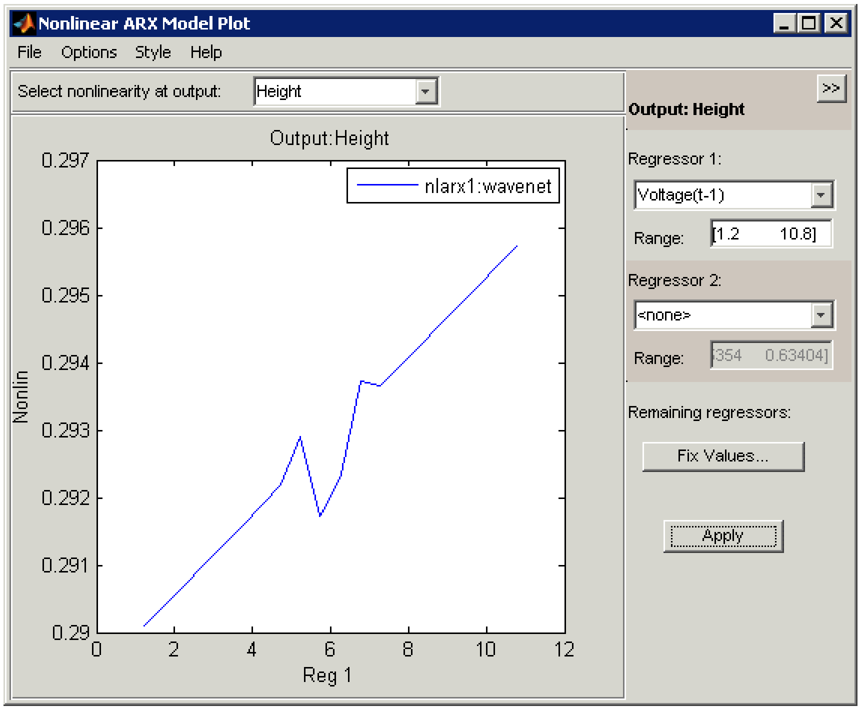
2 In the Nonlinear ARX Model Plot window, keep the default value for **Regressor 1** at `Voltage(t-1)`. Set **Regressor 2** to `Voltage(t-2)`. Click **Apply**.

The relationship between these regressors is nonlinear, as shown in the following plot.



3 To rotate the nonlinearity surface, select **Style > 3D Rotate** and drag the plot to a new orientation.

4 To display a 1–D cross-section for Regressor 1, set Regressor 2 to `none`, and click **Apply**. The following figure shows the resulting nonlinearity magnitude for Regressor 1, which represents the time-shifted voltage signal, `Voltage(t-1)`.



Changing the Nonlinear ARX Model Structure

In this portion of the tutorial, you estimate a nonlinear ARX model after modifying the default input delay and the nonlinearity settings. Typically, you select model orders and delays by trial and error until you get a model that produces an accurate fit to the data.

You must have already estimated the nonlinear ARX model with default settings, as described in "Estimating a Nonlinear ARX Model with Default Settings" on page 6-13.

1 In the Nonlinear Models dialog box, click the **Model Type** tab, and click the **Regressors** tab.

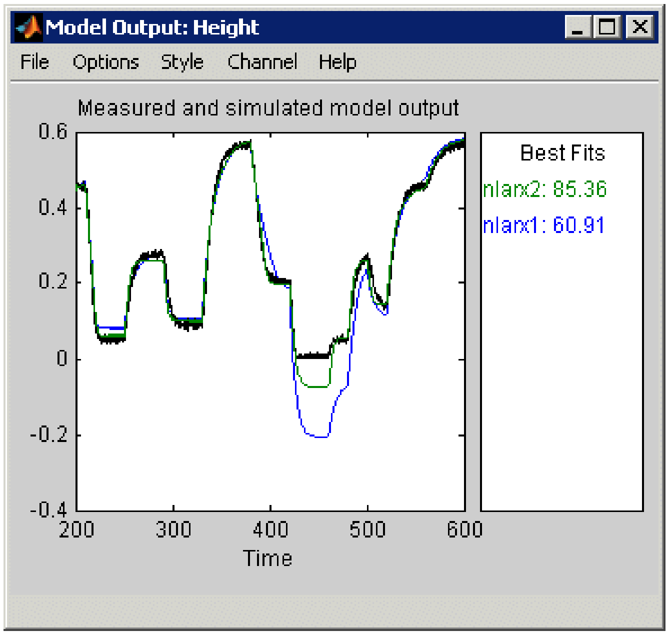
2 For the `Voltage` input channel, double-click the corresponding **Delay** cell, enter `3`, and press **Enter**.

This action updates the **Resulting Regressors** list. The list now includes `Voltage(t-3)` and `Voltage(t-4)`, which are two terms with a minimum input delay of three samples.

3 Click **Estimate**.

This action adds the model `nlarx2` to the System Identification Tool window and updates the Model Output window to include this model. The Nonlinear Models dialog box displays the new estimation information in the **Estimation** tab.

The **Best Fits** area of the Model Output window shows that the `nlarx2` fit is 85.36%.



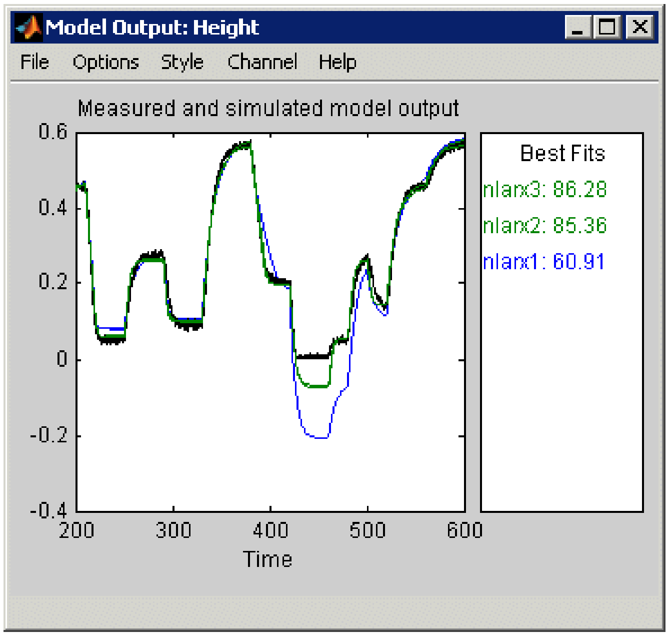
4 In the Nonlinear Models dialog box, click the **Model Properties** tab.

5 In the **Number of units in nonlinear block**, select **Enter**, and type `6`.

6 Click **Estimate**.

This action adds the model `nlarx3` to the System Identification Tool window. It also updates the Model Output window, as shown in the following figure.

The **Best Fits** area of the Model Output window shows that the `nlarx3` fit is 86.28%.



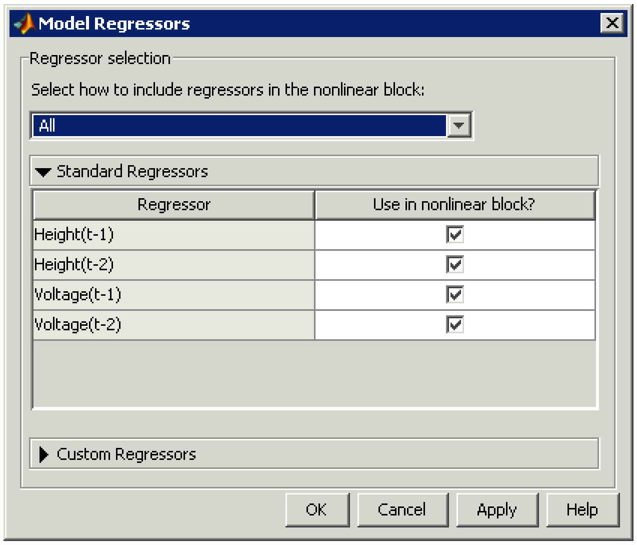
Selecting a Subset of Regressors in the Nonlinear Block

In this portion of the tutorial, you can try to improve the fit by selecting a subset of standard regressors that type as inputs to the nonlinear block. By default, all standard and custom regressors are used in the nonlinear block. In this example, you have only standard regressors.

You must have already specified the model structure, as described in "Changing the Nonlinear ARX Model Structure" on page 6-20.

1 In the Nonlinear Models dialog box, click the **Model Type** tab, and click the **Regressors** tab.

2 Click **Edit Regressors** to open the Model Regressors dialog box.



Select how to include regressors in the nonlinear block: All

Standard Regressors

Regressor	Use in nonlinear block?
Height(t-1)	✓
Height(t-2)	✓
Voltage(t-1)	✓
Voltage(t-2)	✓

3 Clear the following check boxes:

- **Height(t-2)**
- **Voltage(t-1)**

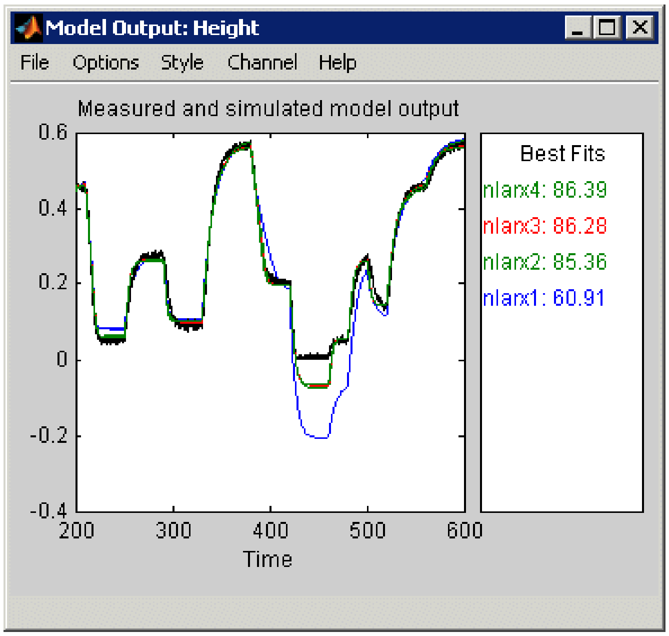
Click **OK**.

This action excludes the time-shifted `Height(t-2)` and `Voltage(t-1)` from the list of inputs to the nonlinear block.

4 Click **Estimate**.

This action adds the model `nlarx4` to the System Identification Tool window. It also updates the Model Output window, as shown in the following figure.

The **Best Fits** area of the Model Output window shows that the `nlarx4` fit is 86.39%, which is only a fraction of a percent improvement from the previous fit.



Changing the Nonlinearity Estimator in a Nonlinear ARX Model

In this portion of the example, you improve the fit of the model estimated with default settings, `nlarx1`, by changing the nonlinearity.

1 In the Nonlinear Models dialog box, click the **Model Type** tab.

2 In the **Initial model** list, select `nlarx1`.

3 Click the **Model Properties** tab.

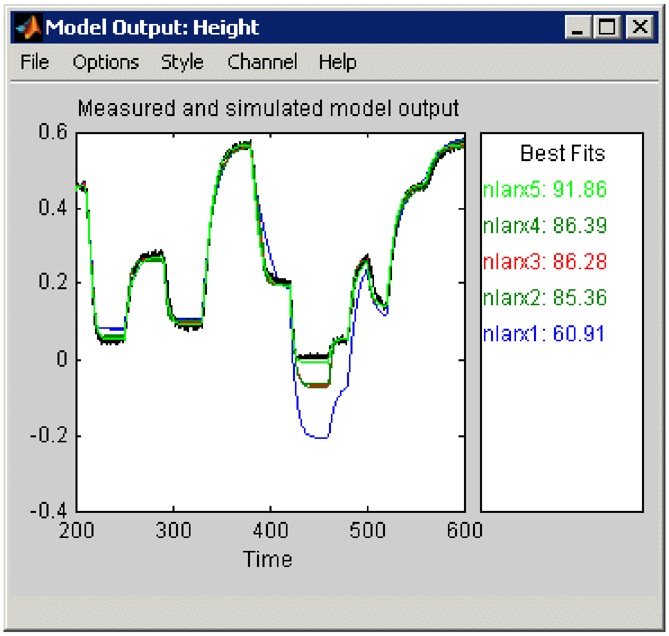
4 In the **Nonlinearity** list, select `Sigmoid Network`.

5 In the **Number of units in nonlinear block** field, type `6`.

6 Click **Estimate**.

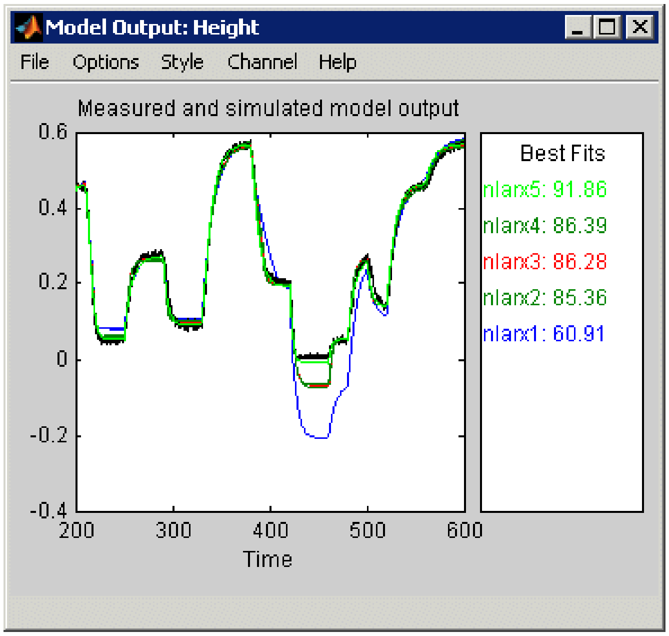
This action adds the model `nlarx5` to the System Identification Tool window. It also updates the Model Output plot, as shown in the following figure.

The **Best Fits** area of the Model Output window shows that the `nlarx5` fit is 91.86%.



Selecting the Best Model

The best model is the simplest model that accurately describes the dynamics. In this tutorial, the best model fit was produced in “Changing the Nonlinearity Estimator in a Nonlinear ARX Model” on page 6-24, as shown in the following figure.


Model Output: Height
File Options Style Channel Help
Measured and simulated model output
0.6
0.4
0.2
0
-0.2
-0.4
200
300
400
500
600
Time
Best Fits
nlarx5: 91.86
nlarx4: 86.39
nlarx3: 86.28
nlarx2: 85.36
nlarx1: 60.91

Estimating Hammerstein-Wiener Models

In this section...

Estimating Hammerstein-Wiener Models with Default Settings

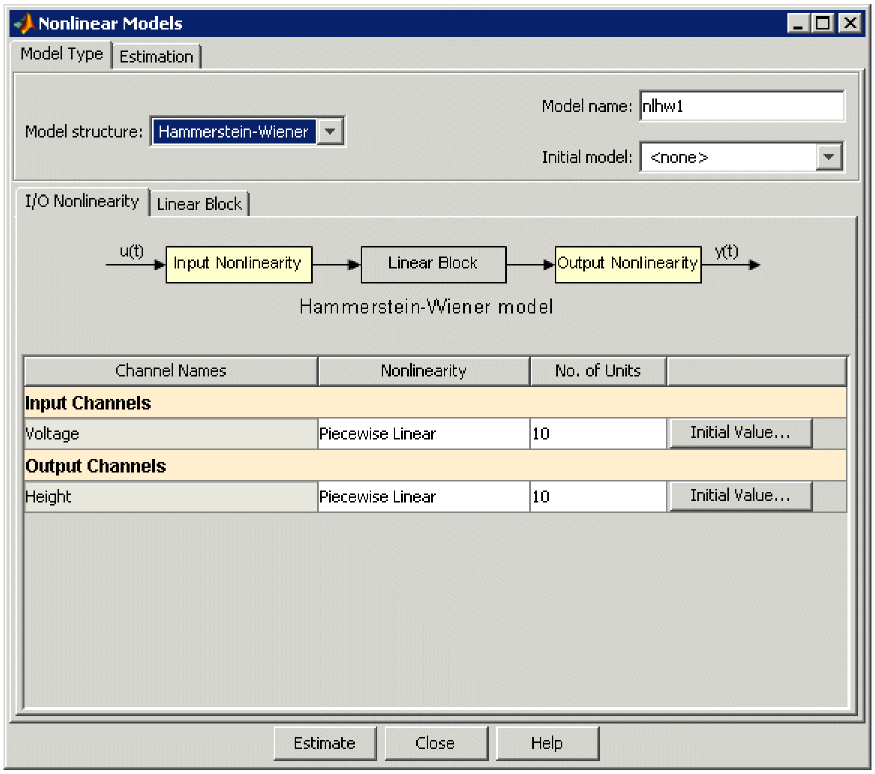
In this portion of the tutorial, you estimate nonlinear Hammerstein-Wiener models using default estimation options.

You must have already prepared the data, as described in "Preparing Data" on page 6-8.

1 In the System Identification Tool window, select **Estimate > Nonlinear models** to open the Nonlinear Models dialog box.

2 In the **Model Type** tab, select `Hammerstein-Wiener` in the **Model Structure** list.

3 Keep the defaults in the **I/O Nonlinearity** tab.



Model structure: Hammerstein-Wiener

Model name: nlhw1

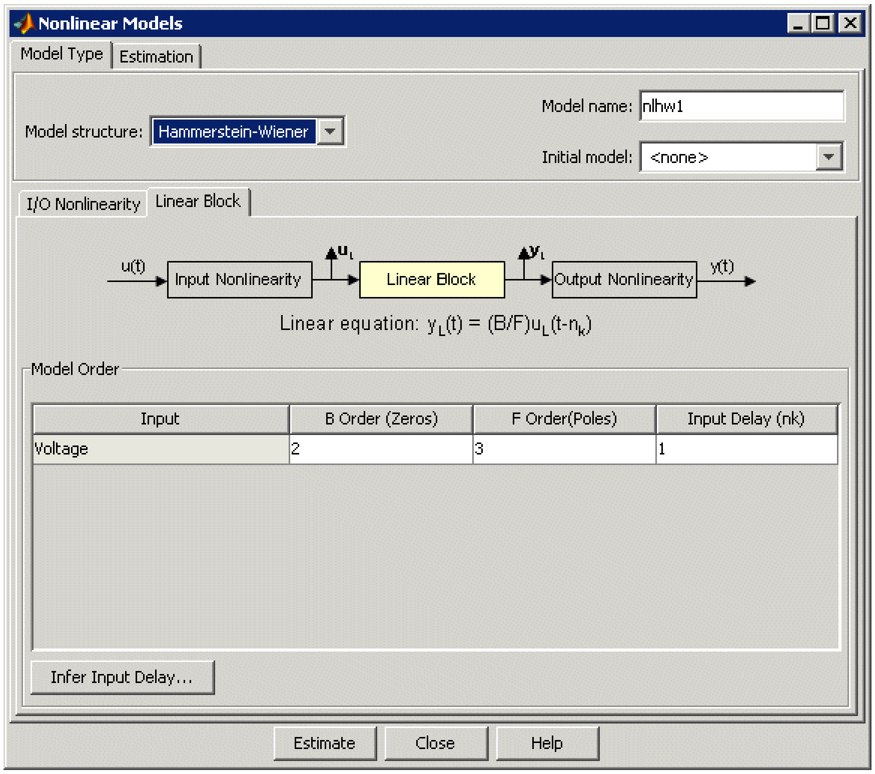
Initial model: <none>

I/O Nonlinearity | Linear Block

Channel Names	Nonlinearity	No. of Units
Input Channels		
Voltage	Piecewise Linear	10
Output Channels		
Height	Piecewise Linear	10

By default, the nonlinearity estimator is `Piecewise Linear` with `10` units for **Input Channels** and **Output Channels**.

4 Keep the defaults in the **Linear Block** tab.



Model structure: Hammerstein-Wiener

Model name: nlhw1

Initial model: <none>

Linear equation: $y_L(t) = (B/F)u_L(t-n_k)$

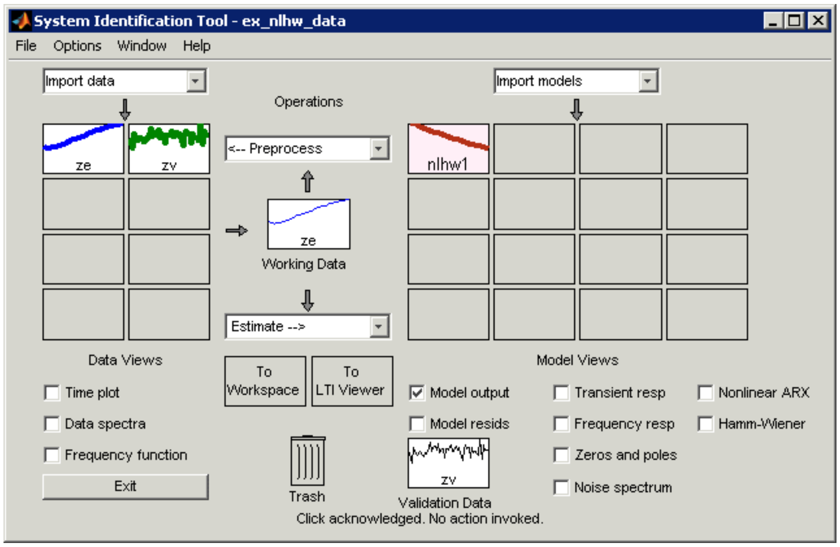
Model Order

Input	B Order (Zeros)	F Order(Poles)	Input Delay (nk)
Voltage	2	3	1

By default, the model orders and delays of the linear output-error (OE) model are n_b=2, n_f=3, and n_k=1.

5 Click **Estimate**.

This action adds the model `nlhw1` to the System Identification Tool window, as shown in the following figure.

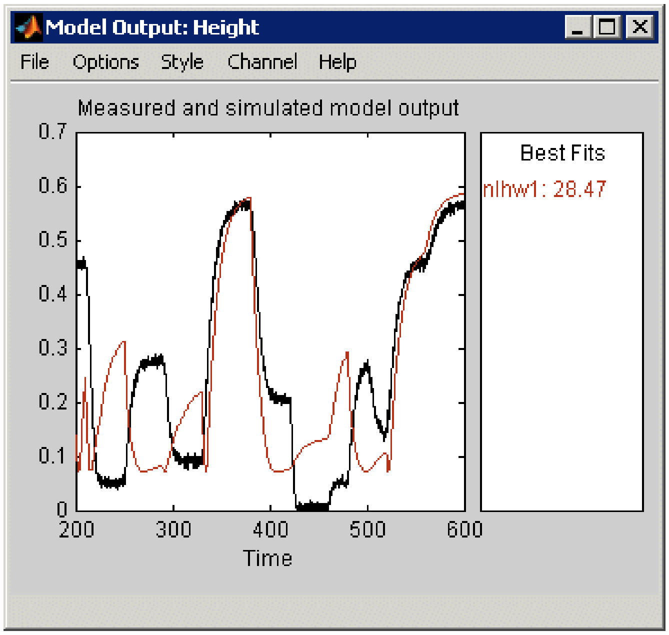


Click acknowledged. No action invoked.

6 In the System Identification Tool window, select the **Model output** check box.

Simulation of the model output uses the input validation data as input to the model. It plots the simulated output on top of the output validation data.

The **Best Fits** area of the Model Output window shows that the agreement between the model output and the validation-data output is 28.47%. Thus, the default settings do not produce an accurate fit.

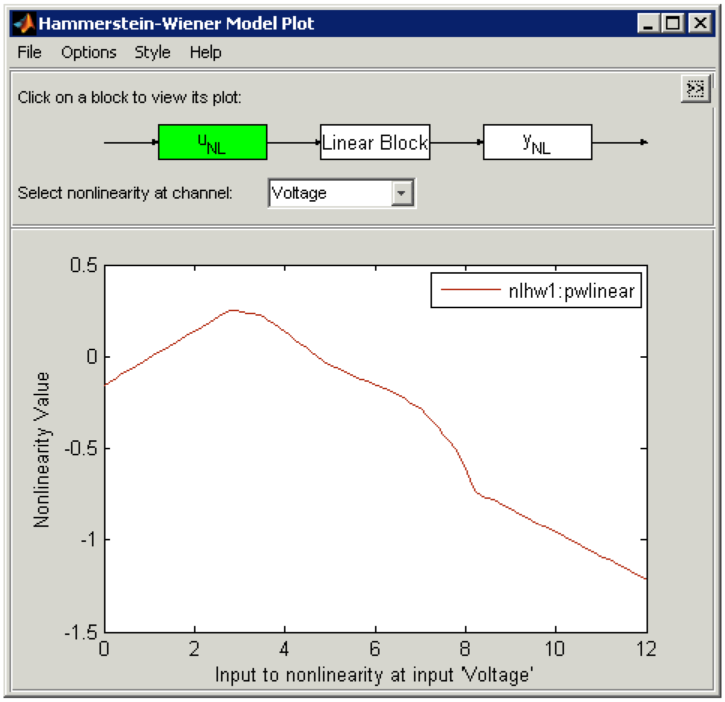


Plotting the Nonlinearities and Linear Transfer Function

You can plot the input/output nonlinearities and the linear transfer function of the model on a Hammerstein-Wiener plot.

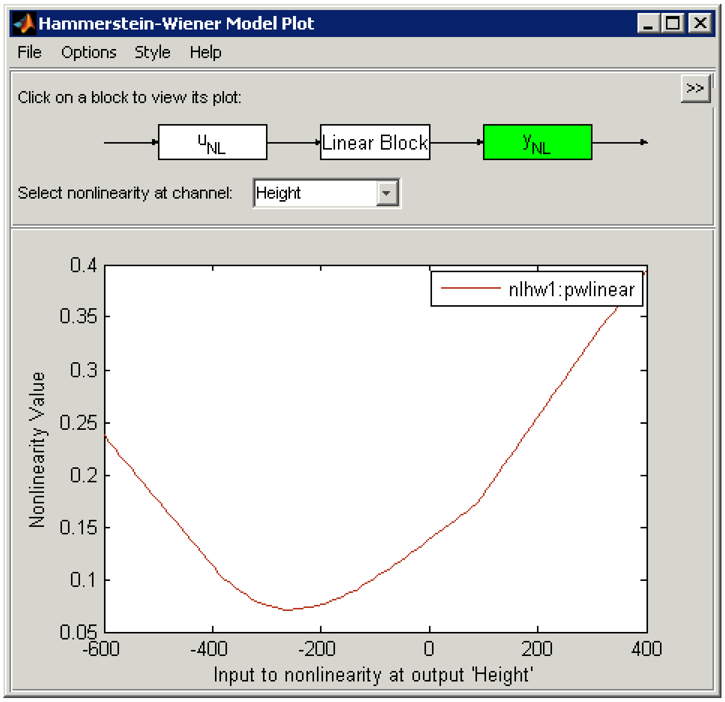
1 In the System Identification Tool window, select the **Hamm-Wiener** check box to view the Hammerstein-Wiener model plot.

The plot displays the input nonlinearity, as shown in the following figure.


Hammerstein-Wiener Model Plot
File
Options
Style
Help
Click on a block to view its plot:
u_{NL}
Linear Block
y_{NL}
Select nonlinearity at channel:
Voltage
nlhw1:pwlinear
Nonlinearity Value
Input to nonlinearity at input 'Voltage'
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10
12

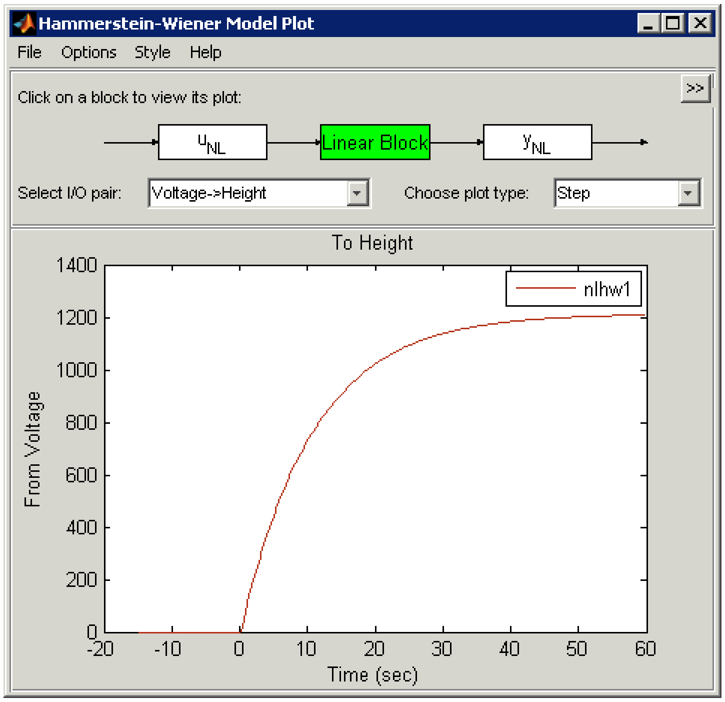
2 Click the $\mathbf{y_{NL}}$ rectangle in the top portion of the Hammerstein-Wiener Model Plot window.

The plot updates to display the output nonlinearity.

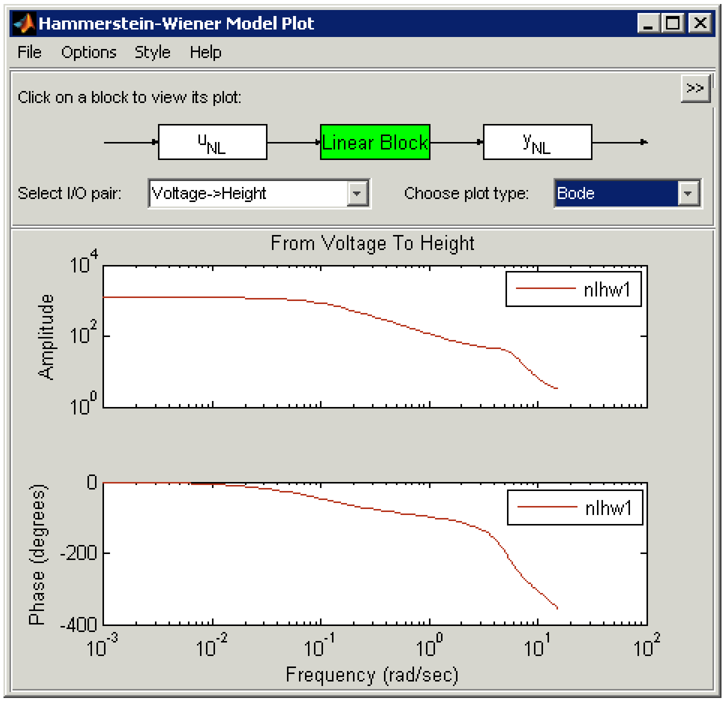

Hammerstein-Wiener Model Plot
File
Options
Style
Help
>>
Click on a block to view its plot:
u_NL
Linear Block
y_NL
Select nonlinearity at channel:
Height
Nonlinearity Value
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
-600
-400
-200
0
200
400
Input to nonlinearity at output 'Height'
nlhw1:pwlinear

3 Click the **Linear Block** rectangle in the top portion of the Hammerstein-Wiener Model Plot window.

The plot updates to display the step response of the linear transfer function.



4 In the **Choose plot type** list, select `Bode`. This action displays a Bode plot of the linear transfer function, as shown in the following figure.



Changing the Hammerstein-Wiener Model Structure

In this portion of the tutorial, you estimate a Hammerstein-Wiener model after modifying the default model order and the nonlinearity settings. Typically, you select model orders and delays by trial and error until you get a model that produces an accurate fit to the data.

You must have already estimated the Hammerstein-Wiener model with default settings, as described in “Estimating Hammerstein-Wiener Models with Default Settings” on page 6-27.

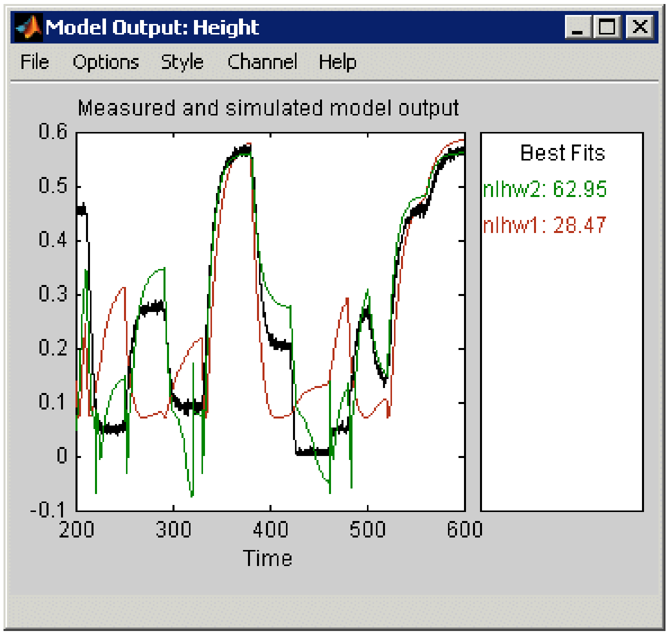
1 In the Nonlinear Models dialog box, click the **Model Type** tab, and click the **Linear Block** tab.

2 For the `Voltage` input channel, double-click the corresponding **Input Delay (nk)** cell, type `3`, and press **Enter**.

3 Click **Estimate**.

This action adds the model `nlhw2` to the System Identification Tool window and the Model Output window is updated to include this model, as shown in the following figure.

The **Best Fits** area of the Model Output window shows that the `nlhw2` fit is 62.95%.



4 In the Nonlinear Models dialog box, select the **I/O Nonlinearity** tab.

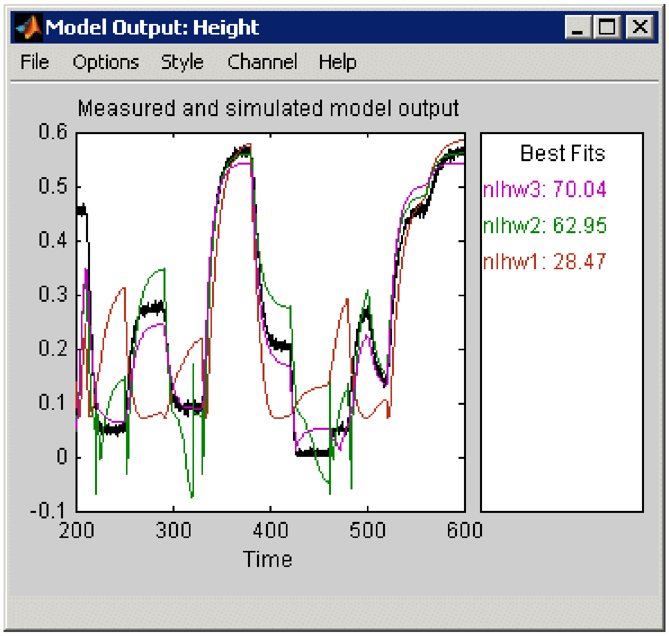
5 For the `Voltage` input channel, double-click the corresponding **No. of Units** cell, and type `20` as the number of units. Press **Enter**.

This action changes the number of nonlinearity units for the `Piecewise Linear` nonlinearity estimator corresponding to the input channel.

6 Click **Estimate**.

This action adds the `nlhw3` model to the System Identification Tool window. It also updates the Model Output window, as shown in the following figure.

The **Best Fits** area of the Model Output window shows that the `nlhw3` fit is 70.04%.



Changing the Nonlinearity Estimator in a Hammerstein-Wiener Model

In this portion of the example, you improve the fit by changing the nonlinearity estimator.

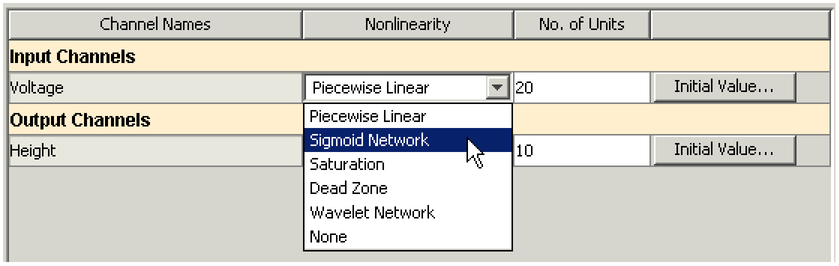
1 In the Nonlinear Models dialog box, click the **Model Type** tab, and click the **Linear Block** tab.

2 For the `Voltage` input channel, double-click the corresponding **Input Delay (nk)** cell, type `1`, and press **Enter**.

This action restores the input delay to the default value.

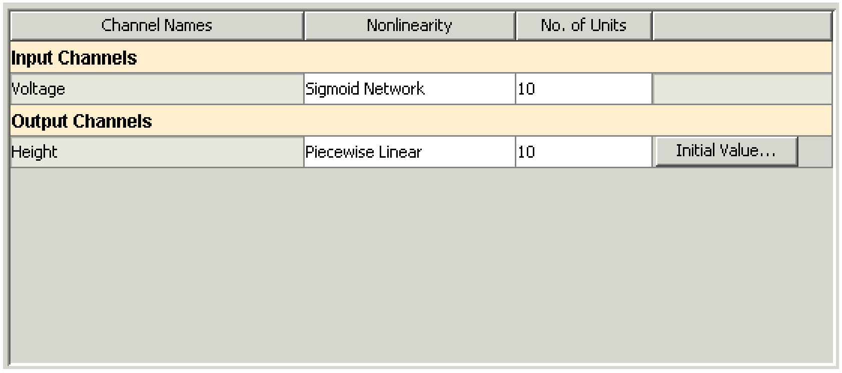
3 In the Nonlinear Models dialog box, click the **Model Type** tab, and click the **I/O Nonlinearity** tab.

4 For the `Voltage` input, click the **Nonlinearity** cell, and select `Sigmoid Network` from the list, as shown in the following figure.



Channel Names	Nonlinearity	No. of Units	
Input Channels			
Voltage	Piecewise Linear	20	Initial Value...
Output Channels			
Height		10	Initial Value...

This action updates the corresponding **No. of Units** cell to 10 sigmoid units, as shown in the following figure.

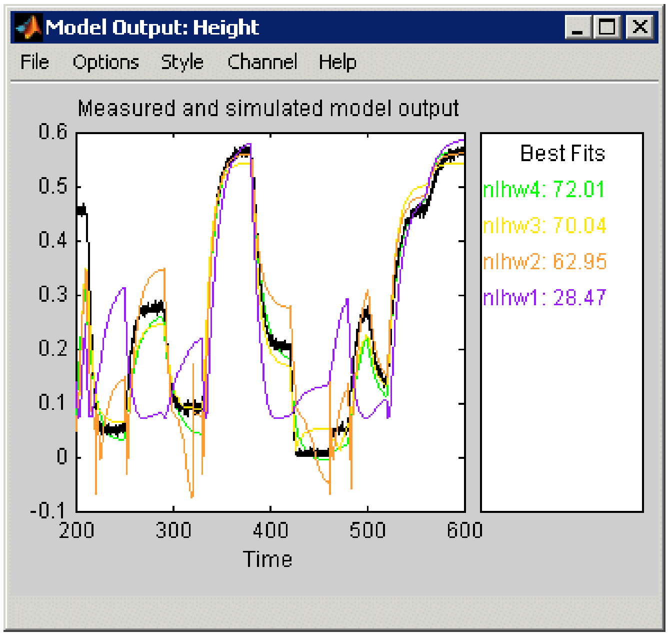


Channel Names	Nonlinearity	No. of Units	
Input Channels			
Voltage	Sigmoid Network	10	
Output Channels			
Height	Piecewise Linear	10	Initial Value...

5 Click **Estimate**.

This action adds the model `nlhw4` to the System Identification Tool window. It also updates the Model Output window, as shown in the following figure.

The **Best Fits** area of the Model Output window shows that the `nlhw4` fit is 72.01%.

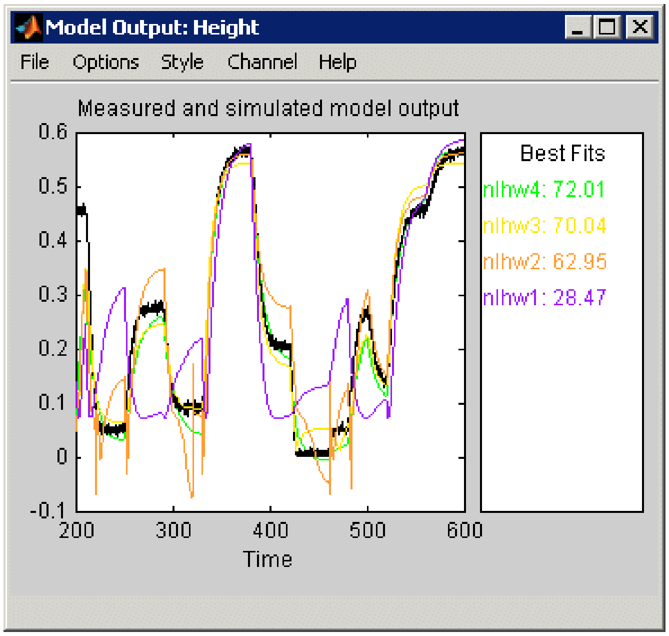


Tip If you know that your system includes saturation or dead-zone nonlinearities, you can specify these specialized nonlinearity estimators in your model. `Piecewise Linear` and `Sigmoid Network` are nonlinearity estimators for general nonlinearity approximation.

Selecting the Best Model

The best model is the simplest model that accurately describes the dynamics.

In this example, the best model fit was produced in “Changing the Nonlinearity Estimator in a Hammerstein-Wiener Model” on page 6-37, as shown in the following figure.



Index